Accurate Prediction of Worst Case Eye Diagrams for Non-Linear Signaling Systems

Aadithya V. Karthik*, Sayak Ray, Robert Brayton, and Jaijeet Roychowdhury

EECS Dept., The University of California, Berkeley

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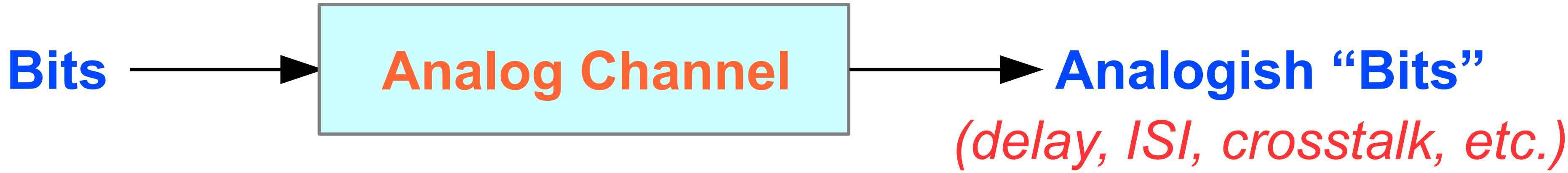
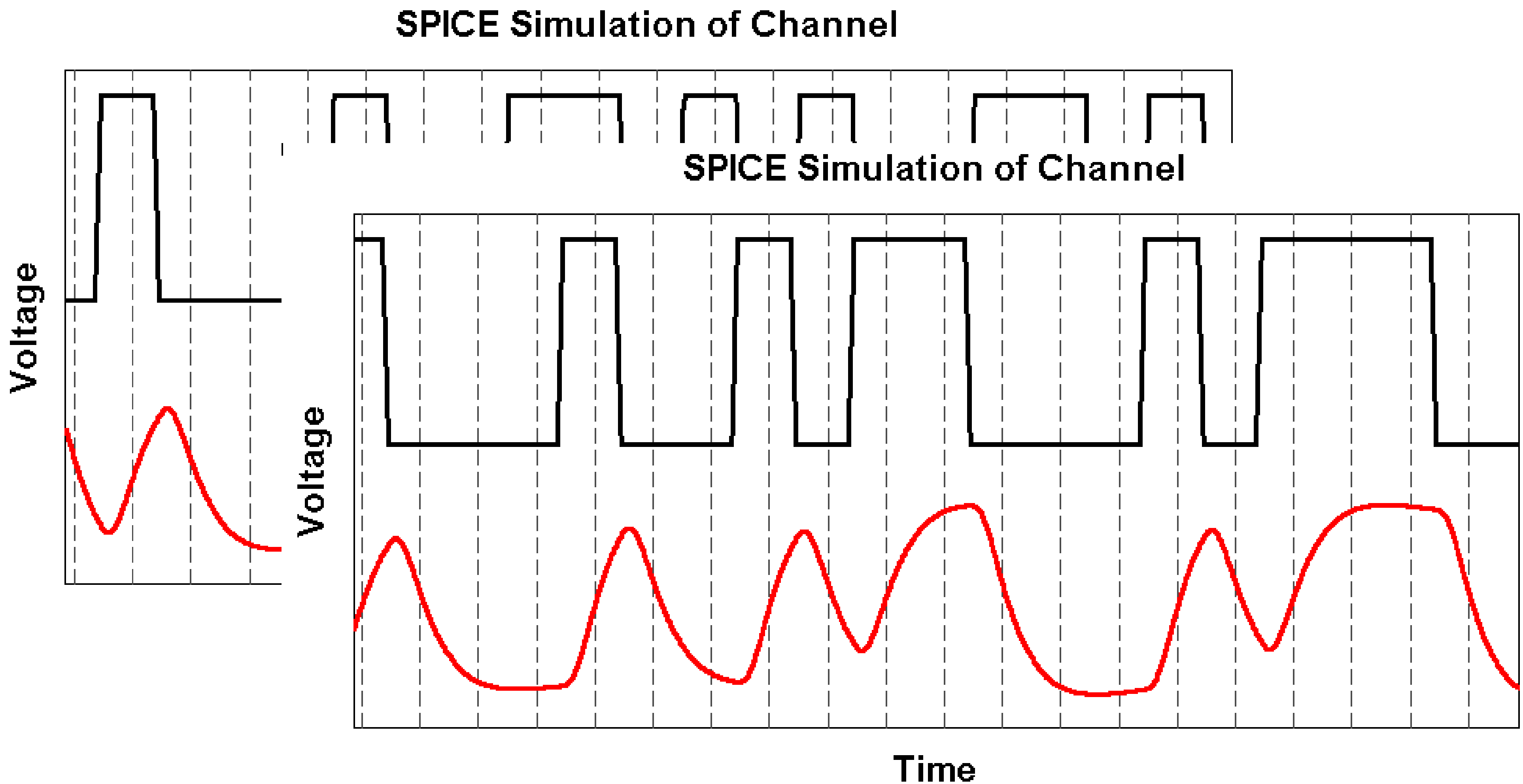
Overview of this talk

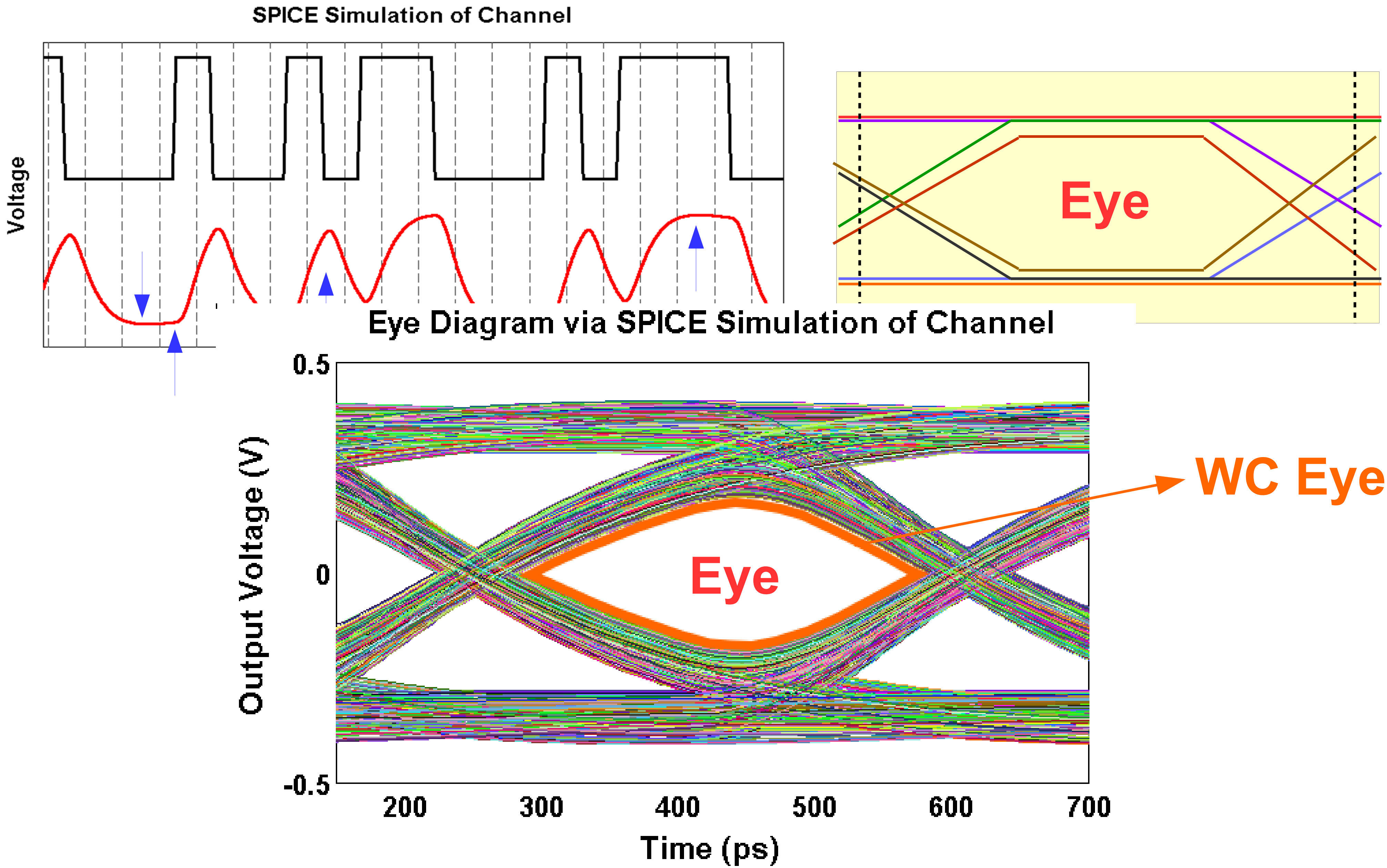
- The Worst Case (WC) eye diagram problem
  - Starting from the basics, *i.e.*, what is an eye diagram?
- Existing algorithms for WC eye estimation
  - PDA, illustrated with an example
- Where PDA fails
  - Cannot handle general formulations of problem
- A new algorithm for WC eye computation
  - Illustrated with an example
- Results
  - 8b/10b encoder (PCI Express, USB, etc.)
  - *Our technique is much less pessimistic than PDA*

What is an Eye Diagram (1/2)?



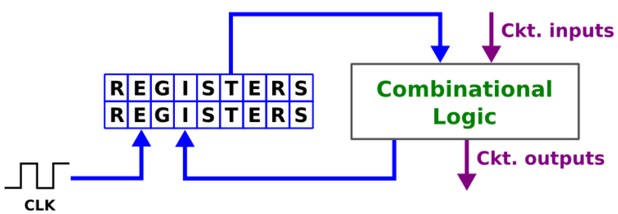


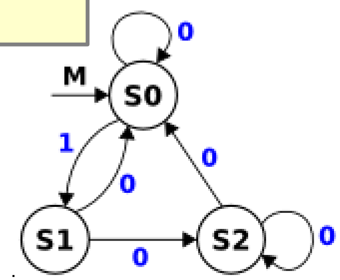
What is an Eye Diagram (2/2)?



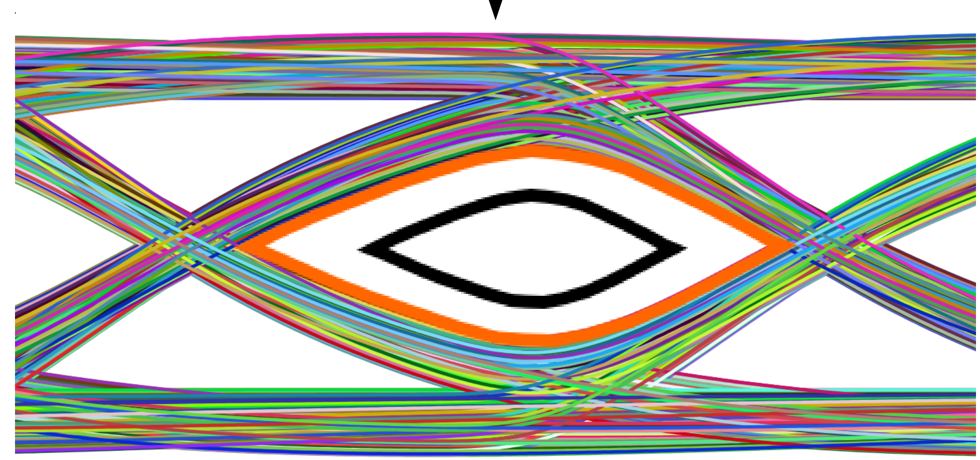
The Worst Case Eye Problem

Digital System → *Arbitrary* / *Correlated* **Bit sequence** → **Analog (LTI) Channel**





~~PDA~~ **Output eye**



Problem: Compute worst-case eye

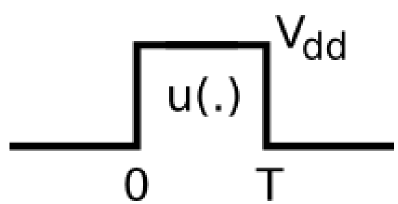
- Pure analog → PDA
- Analog + Digital
  - Non-Linear System
  - Correlated bits
  - PDA too pessimistic
  - **Our new algorithm!**

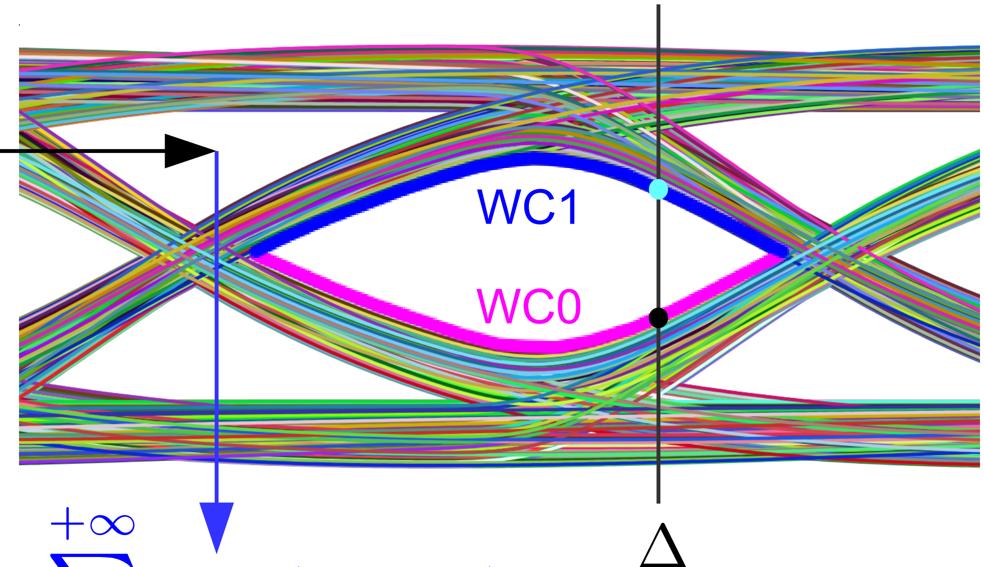
Peak Distortion Analysis (PDA)

- Assume channel is LTI
- Key idea: WC Eye = 2 Optimization Problems

$\{\ldots, b_{-1}, b_0, b_1, \ldots\}$ → **LTI Channel** →

$$\sum_{k=-\infty}^{+\infty} b_k u(t-kT)$$





$$\sum_{k=-\infty}^{+\infty} b_k h(t-kT)$$

Linear combination of the bits!

$$\text{Eg. } \frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1$$

$b_0 = 1$
$b_{-3} = b_{-1} = 0$
$b_{-2} = b_1 = 1$

Need mutually independent bits [0, 1, 0, 1, 1]

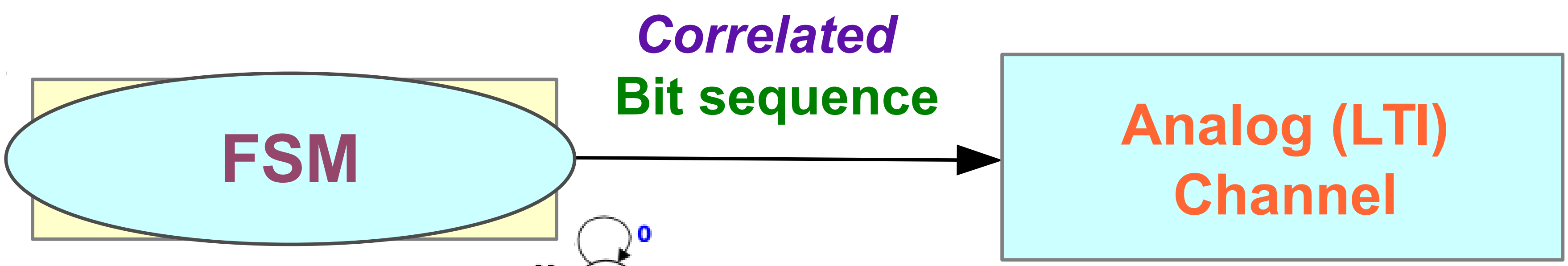
$$\text{WC1: minimize} \sum_{k=-M}^{\lfloor \Delta/T \rfloor} b_k h(\Delta - kT)$$

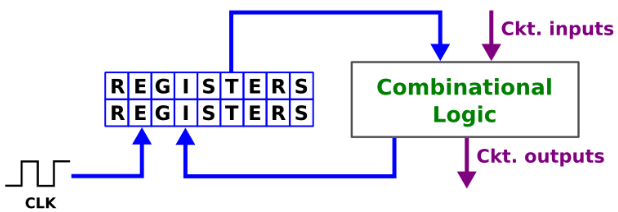
$(\text{subj. to } b_0 = 1)$

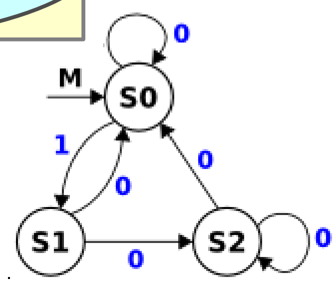
$$\text{WC1}(\Delta) = 0.5$$

Correlated bits: PDA fails!

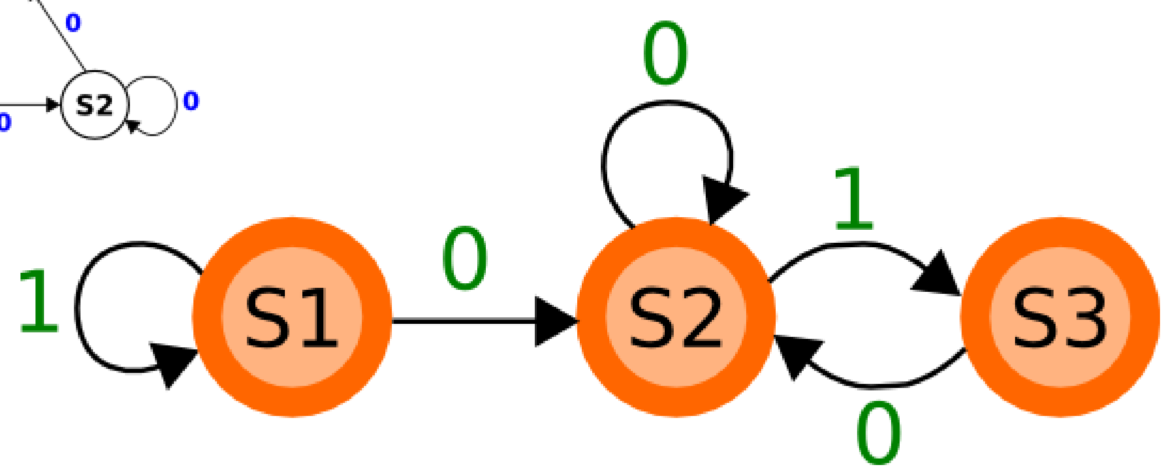
FSMs for Modeling Correlated Bits







Arbitrary digital logic, arbitrary bit correlations



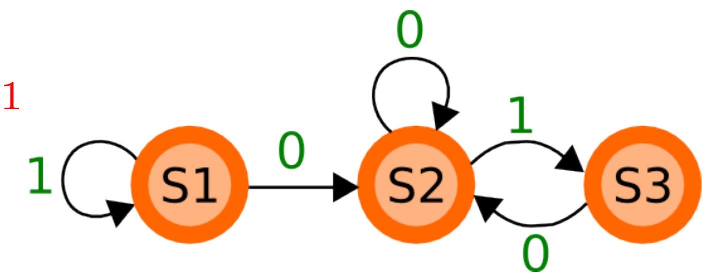
- Finite number of states
- Arcs denoting state transitions
  - Each arc has an output bit

For example, this FSM can never produce the sequence [0, 1, 1]

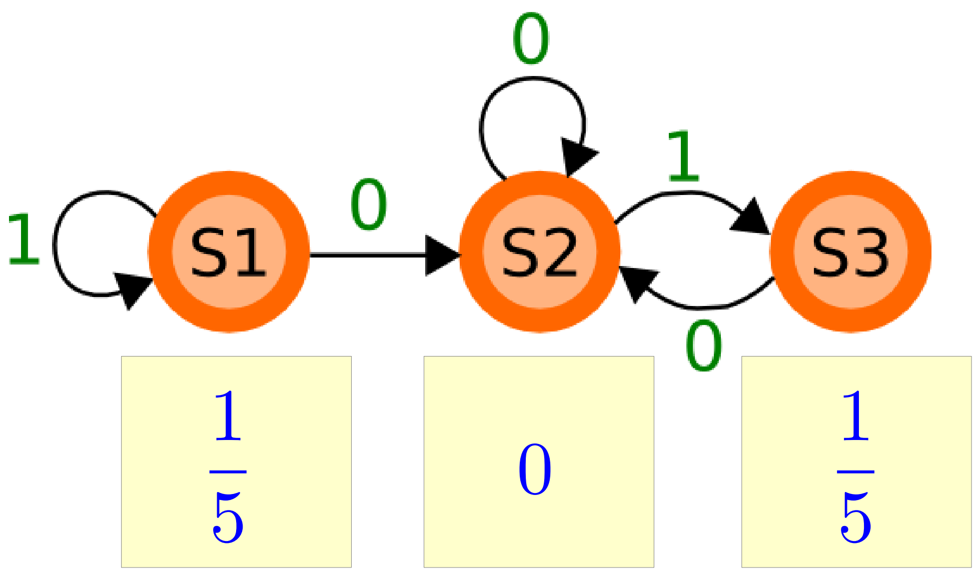
Algorithm for Correlated WC Eye

Example. Minimize $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1$

subj. to $b_0 = 1$, $\{b_k\}$ comes from FSM



Key idea: Best partial sum ending in state Si



Best $\frac{1}{5}b_{-3}$

Algorithm for Correlated WC Eye

Example. Minimize $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1$

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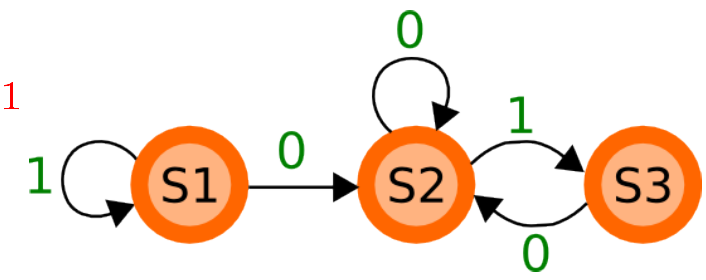

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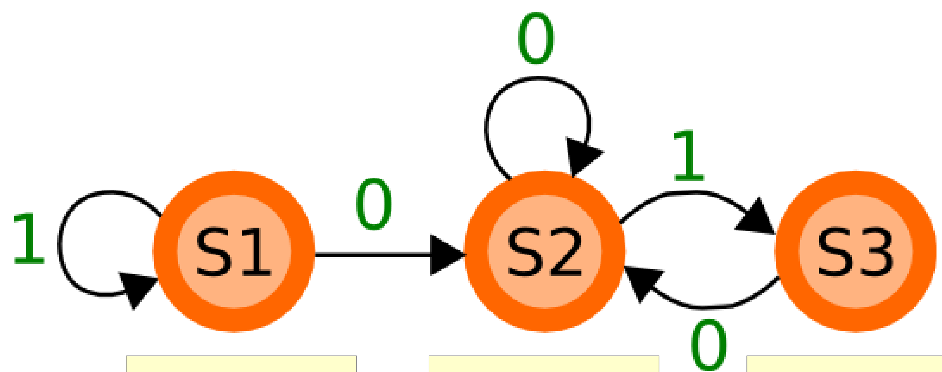

Best $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2}$

$\frac{1}{5}$	0	$\frac{1}{5}$
$\frac{1}{5} - \frac{1}{4}$	$0 + 0$	$0 - \frac{1}{4}$

Algorithm for Correlated WC Eye

Example. Minimize $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1$

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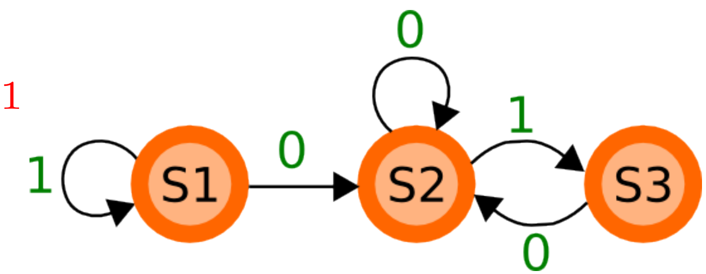
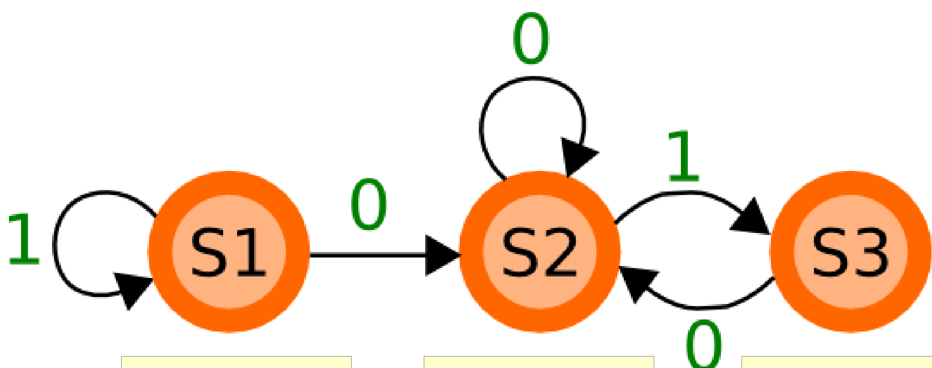


Key idea: Best partial sum ending in state Si



S1	S2	S3
$-\frac{1}{20}$	0	$-\frac{1}{4}$
$-\frac{1}{20}+\frac{1}{2}$	$-\frac{1}{4}+0$	$0+\frac{1}{2}$

Best $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1}$

Algorithm for Correlated WC Eye

Example. Minimize $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1$

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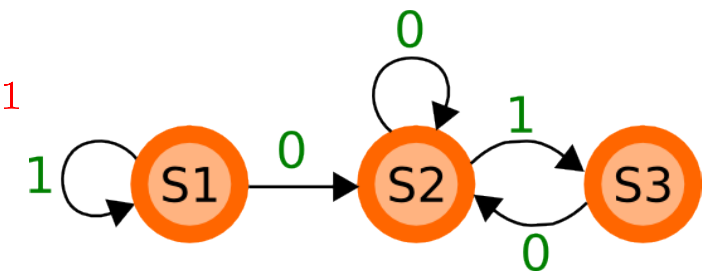

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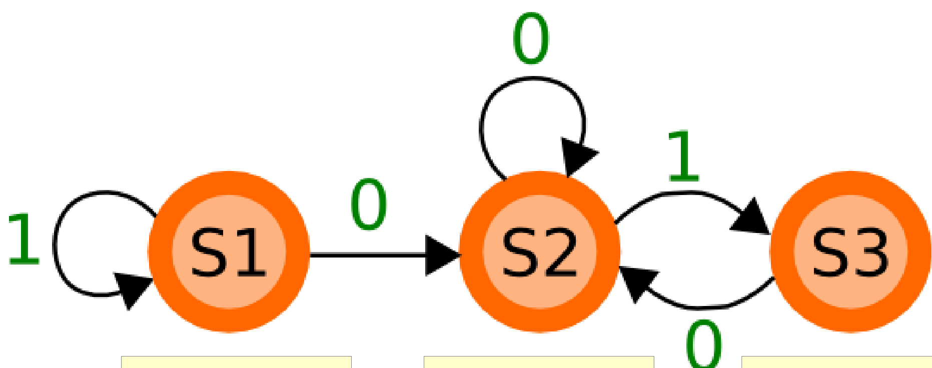

$\frac{9}{20}$	$-\frac{1}{4}$	$\frac{1}{2}$
$\frac{9}{20}+1$	✗	$-\frac{1}{4}+1$

Best $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0$

Algorithm for Correlated WC Eye

Example. Minimize $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1$

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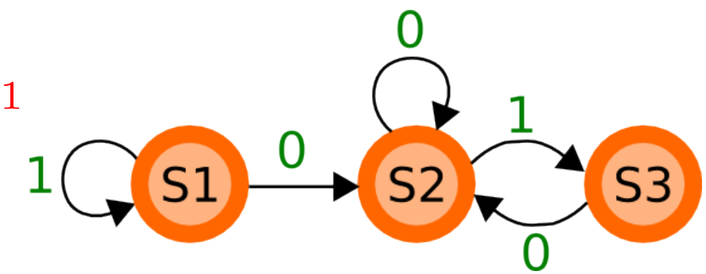

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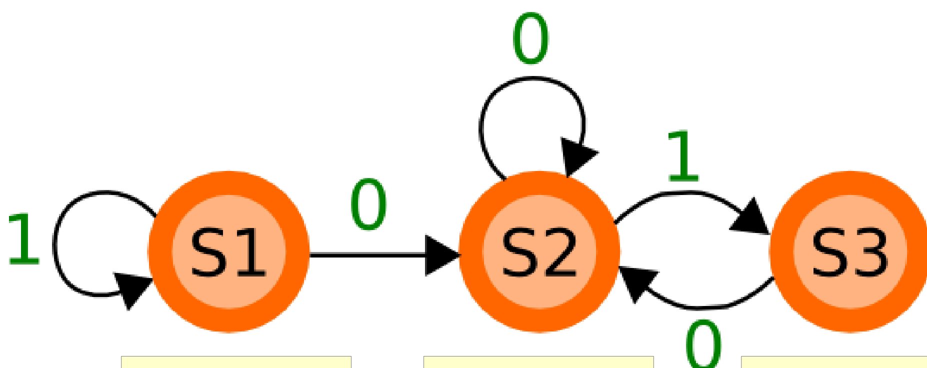


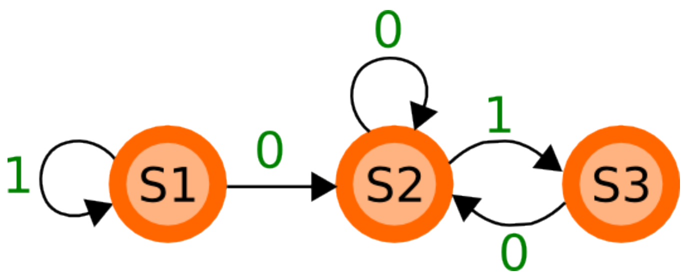
$\frac{29}{20}$	×	$\frac{3}{4}$
$\frac{29}{20} - \frac{1}{4}$	$\frac{3}{4} + 0$	×

Best $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1$

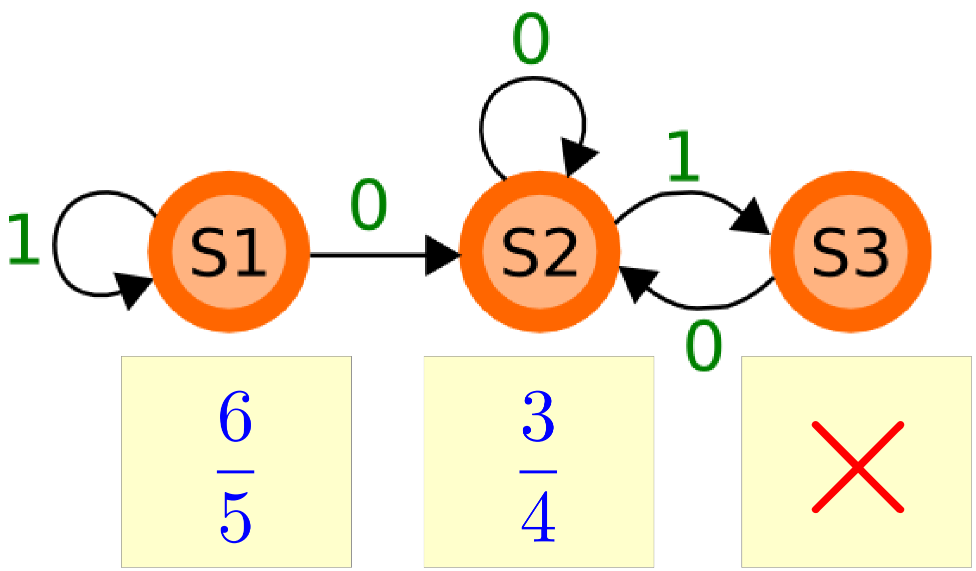
Algorithm for Correlated WC Eye

Example. Minimize $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1$

subj. to $b_0 = 1$, $\{b_k\}$ comes from FSM



Key idea: Best partial sum ending in state Si



Dynamic programming

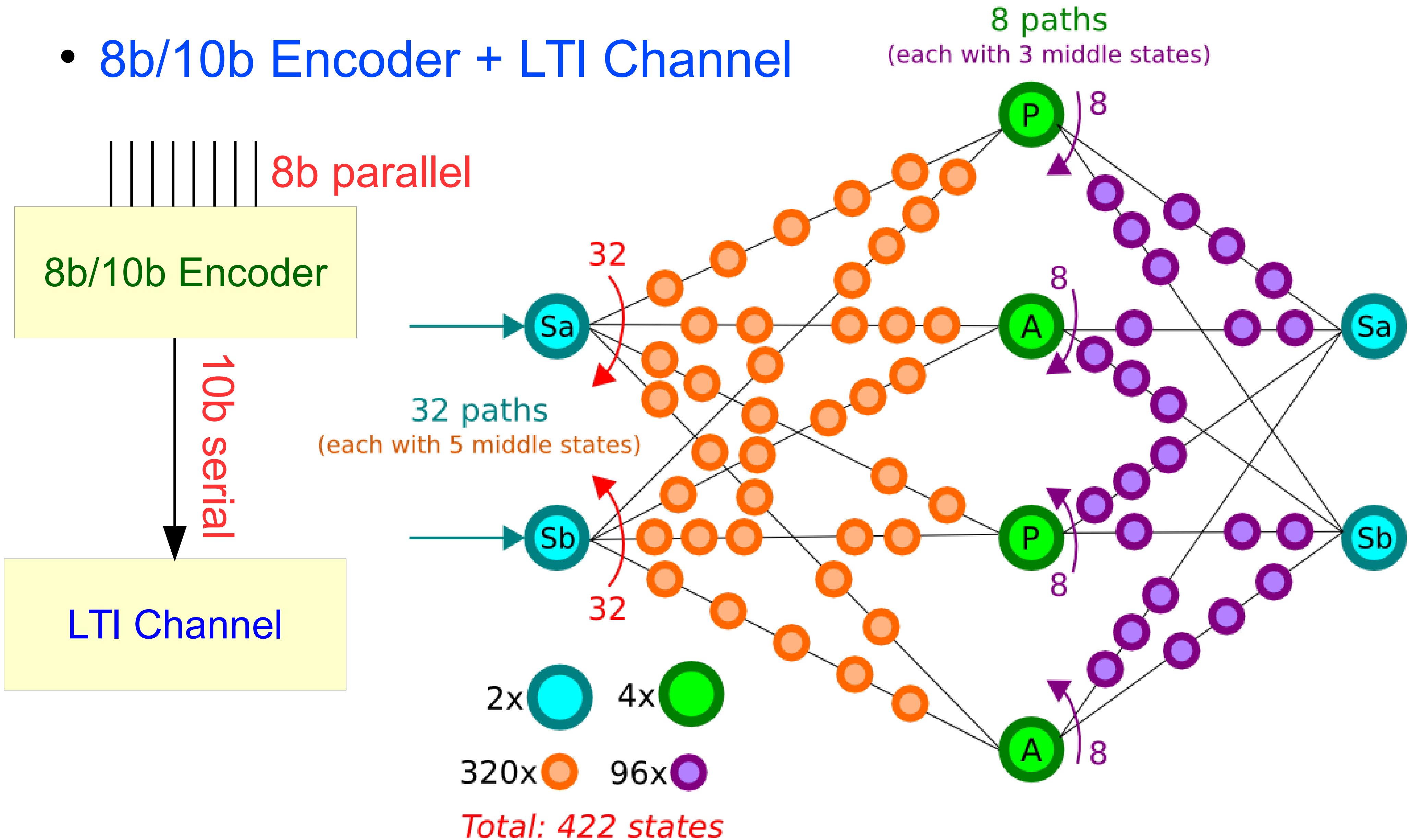
Best $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1$

$$= \mathrm{WC1}(\Delta) = \frac{3}{4} = 0.75$$

Compare to PDA, which pessimistically predicts 0.5

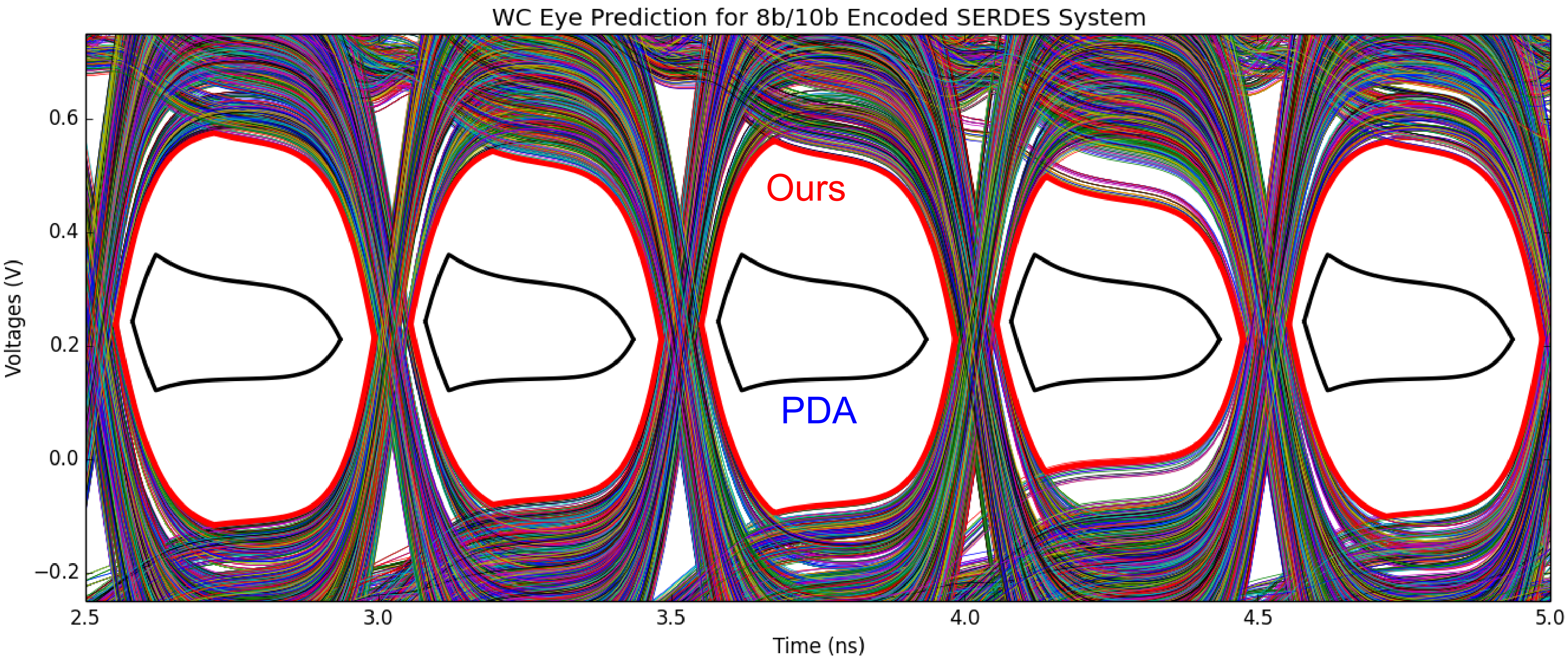
Results: 8b/10b Encoder (1/2)

- 8b/10b Encoder + LTI Channel



Results: 8b/10b Encoder (2/2)

- 8b/10b Encoder + LTI Channel



Summary

- WC eye computation is important
- Traditional PDA cannot handle bit correlations
- Our new technique can
- Key ideas behind our technique
  - Model bit correlations as FSMs
  - Reduce WC eye computation to an optimization problem
  - Use dynamic programming to solve the above efficiently
- Results
  - (7, 4) Hamming code
  - 8b/10b Encoder
- Future work
  - Deterministic worst case → Probabilistic distributions

Questions