Variation Aware Cross-Talk Aggressor Alignment by Mixed Integer Linear Programming

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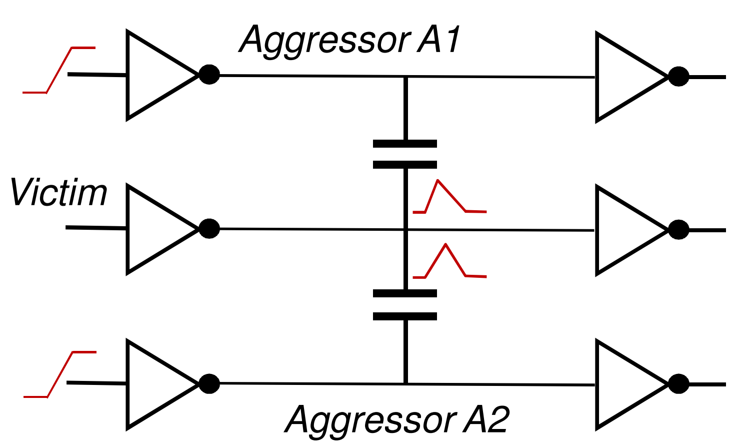
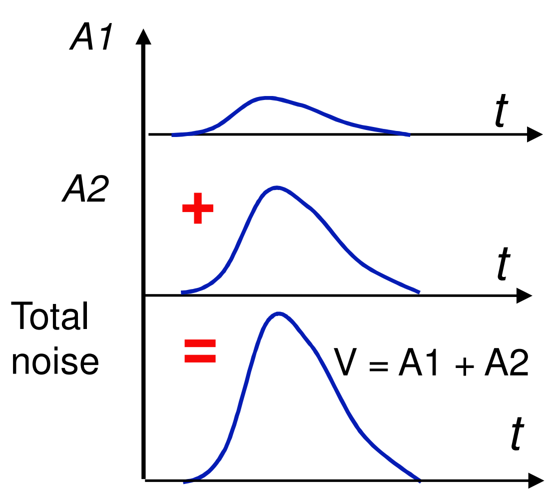
Cross Coupling Noise

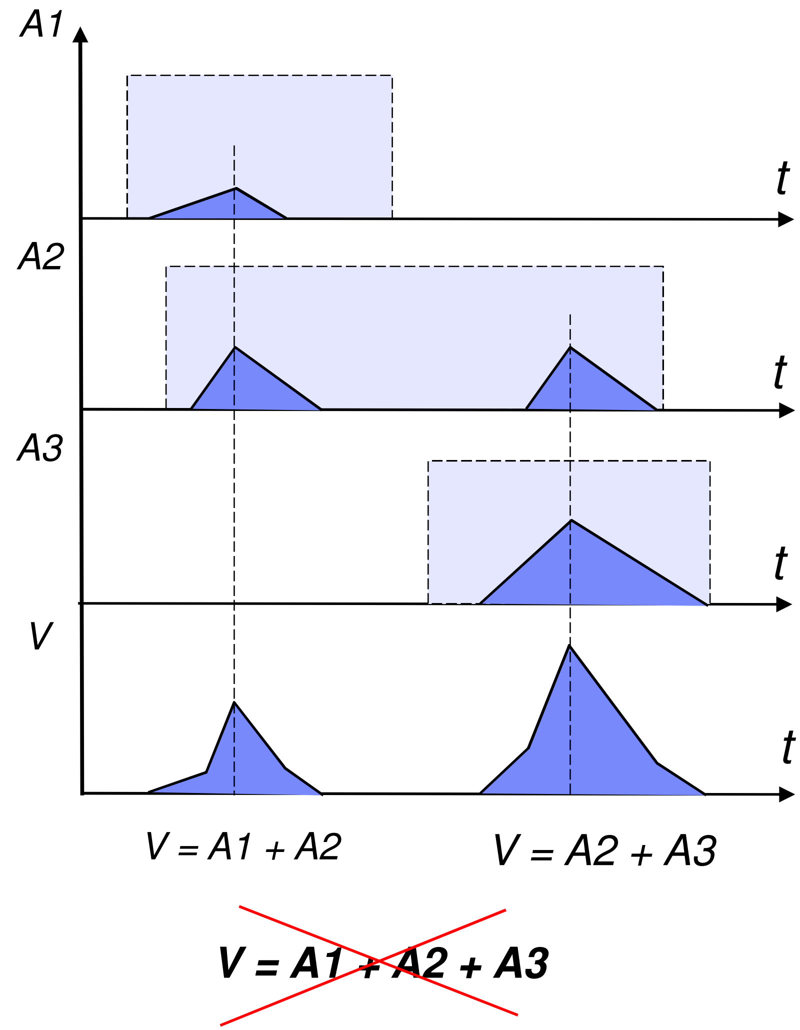




- **Aggressor nets inject noise pulses into victim net through coupling capacitances**
- **Noise pulses can affect both state and transition of victim net causing functional and timing failures**
- **Conservative noise analysis:**
  - All aggressor nets switch simultaneously
  - All aggressor noise pulses are combined
    - Typically by linear superposition
  - Can be too pessimistic
    - Because of neglecting circuit timing prohibiting simultaneous signal transitions

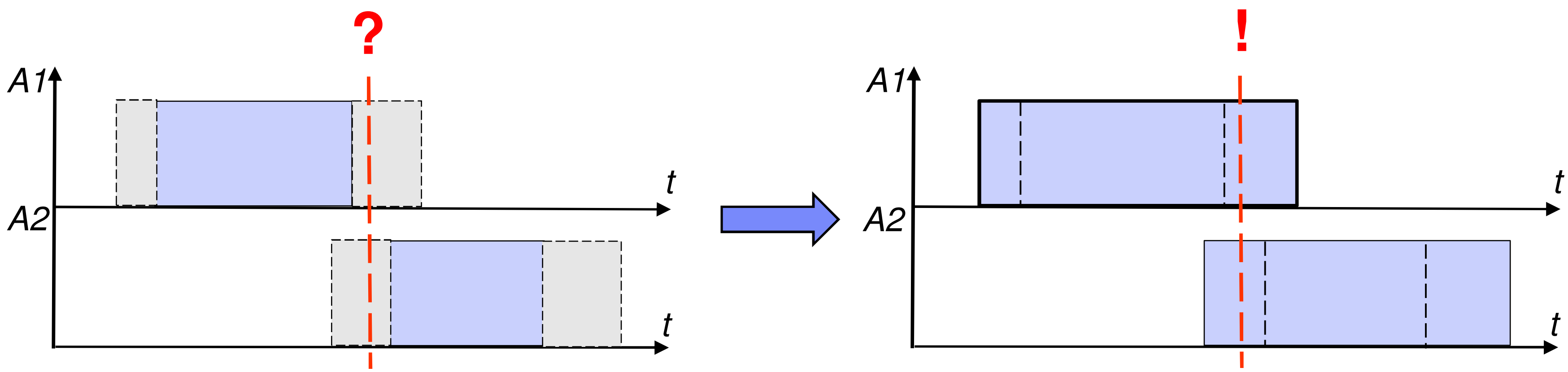
Cross-talk Aggressor Alignment

- **Timing predicts EARLY and LATE signal arrival times**
- **Noise pulses occur only inside timing windows**
  - Timing windows are defined by their start and end moments
- **Only pulses of overlapping timing windows can be combined for total noise**
  - Potential source of pessimism reduction
- **Noise analysis computes combination of overlapping timing windows resulting in worst noise pulse**
  - Well known sweeping line algorithms solves this problem



Aggressor Alignment under Process Variation

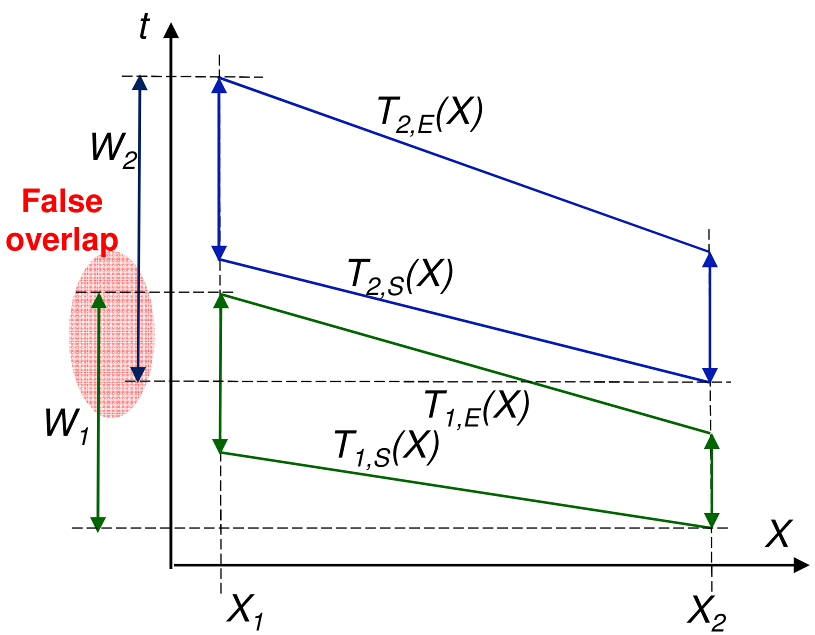
- **Process and environmental variations cause variability of timing windows**
  - Sometimes it is not clear if windows align at some values of parameters

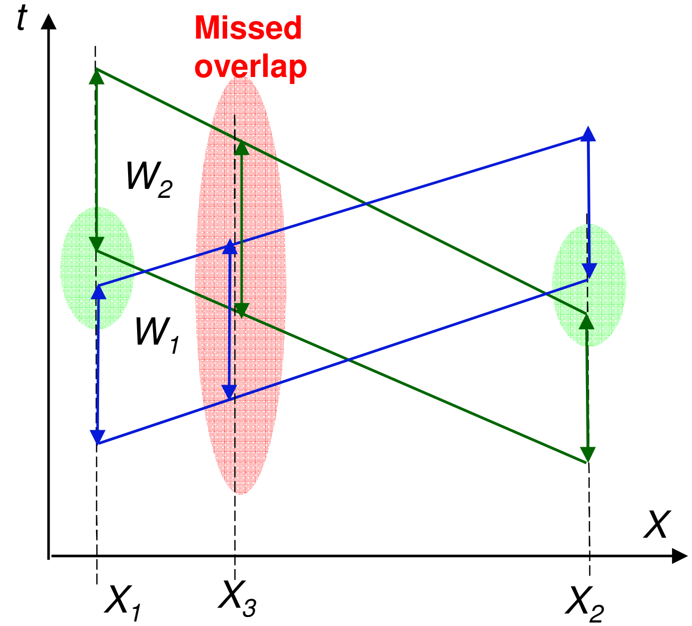


- **Conservative approximation expands timing windows**
- **It can be too pessimistic leading to overestimation of the combined noise**
  - Neglects that variations of start and end moments of windows are highly correlated
    - For example higher Vdd makes most transitions occur earlier
- **There were several attempts to solve this problem**
  - Methods were too complex for implementation, not general, inefficient and inaccurate for industrial applications
- **Even full corner enumeration can be too optimistic, missing window overlap**

Examples of variational timing windows

- **Timing windows depend on one variational parameter:** $\boldsymbol{T = T_0 + aX}$





- **Conservative window expansion predicts false overlap**

- **Full corner enumeration fails to predict overlap of timing windows**
  – Windows overlap only between process corners

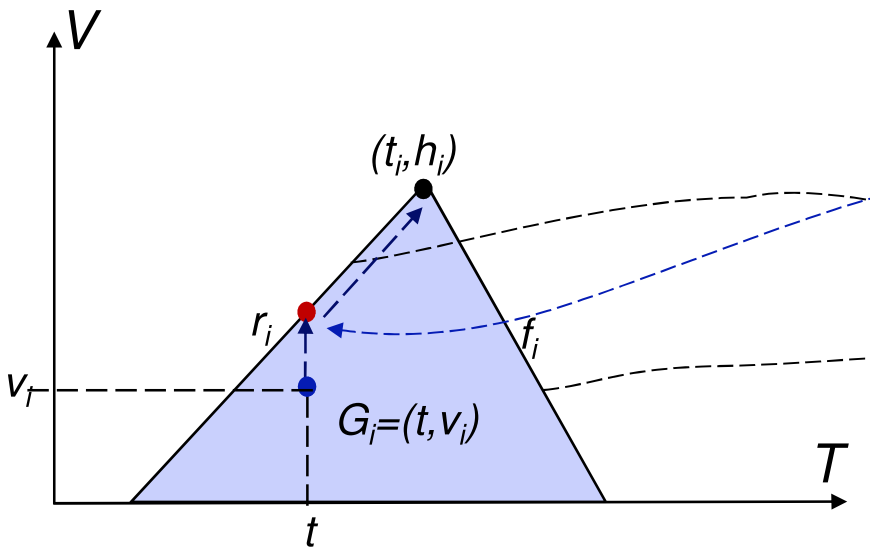
Main Assumptions and Plan of Solution

- **Variability is bounded with min/max corners** $X_{j,min} \le X_j \le T_{j,max}$
  - Non-statistical approach
- **Timing variability is linear function of variational parameters**

$$T = T_0 + \sum_{j=1}^{m} a_j X_j$$

- **Linear program for maximum voltage of single noise pulse**
- **Linear Program (LP) for deterministic aggressor alignment**
  - Fails if not all timing windows intersect
- **Mixed Integer Linear Program (MILP) for deterministic alignment**
- **MILP for variational aggressor alignment**
- **Analysis of efficiency and methods of its improvement**
- **Extension of MILP formulation to:**
  - Victim sensitivity window
  - Non-triangle noise pulses
  - Aggressor switching constraints
  - Discontinuous timing windows

Formulation for Single Noise Pulse

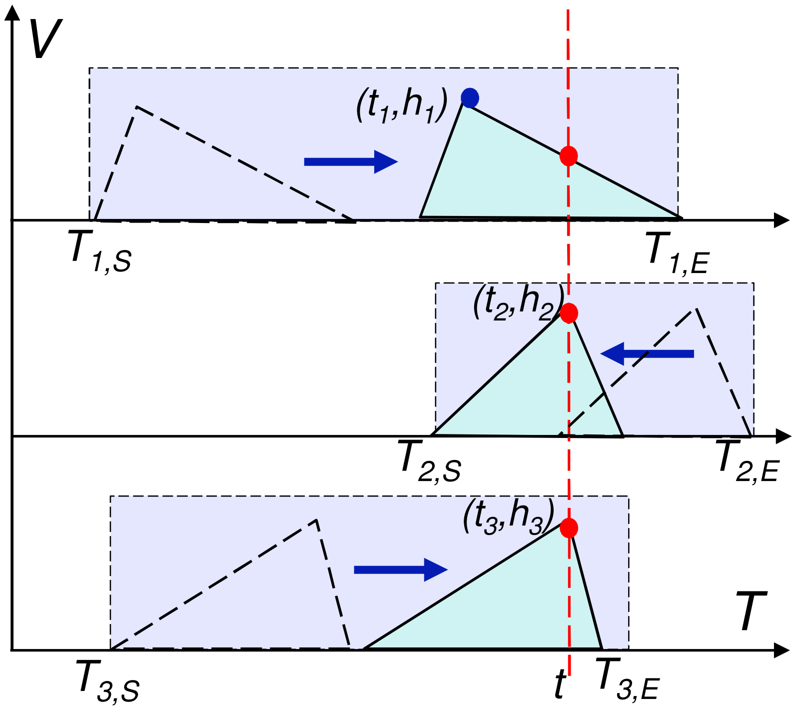


Maximize $\underset{t}{v_i}$ *subject to*

$$(v_i - h_i) - r_i(t - t_i) \le 0$$
$$(v_i - h_i) - f_i(t - t_i) \le 0$$

- **Noise pulse is defined with**
  - its tip point (t_i, v_i)
  - its rising and falling slews: r_i and f_i
- **Point $G_i=(t,v_i)$ measures possible voltage due to *i-th* noise pulse at time t**
- **Linear inequalities constrain position of point G_i under rising and falling slopes of noise pulse**
- **Maximization of v_i moves point up to the rising or falling slope of the pulse**

LP for Deterministic Aggressor Alignment



- **Shifts moments t_i of noise pulses for their alignment to maximize total noise**
- **Moves measurement t time to time moment with maximum total noise**
- **Restrict maximum total noise measurement to be taken inside noise pulses**
- **Require individual noise values to be positive**
- **Restrict noise pulses to stay inside timing windows**

$$\underset{t,\, t_i,\, v_i}{Maximize} \quad \sum_{i=1}^{N} v_i \qquad subject\ to$$

$$(v_i - h_i) - r_i(t - t_i) \le 0$$

$$(v_i - h_i) - f_i(t - t_i) \le 0$$

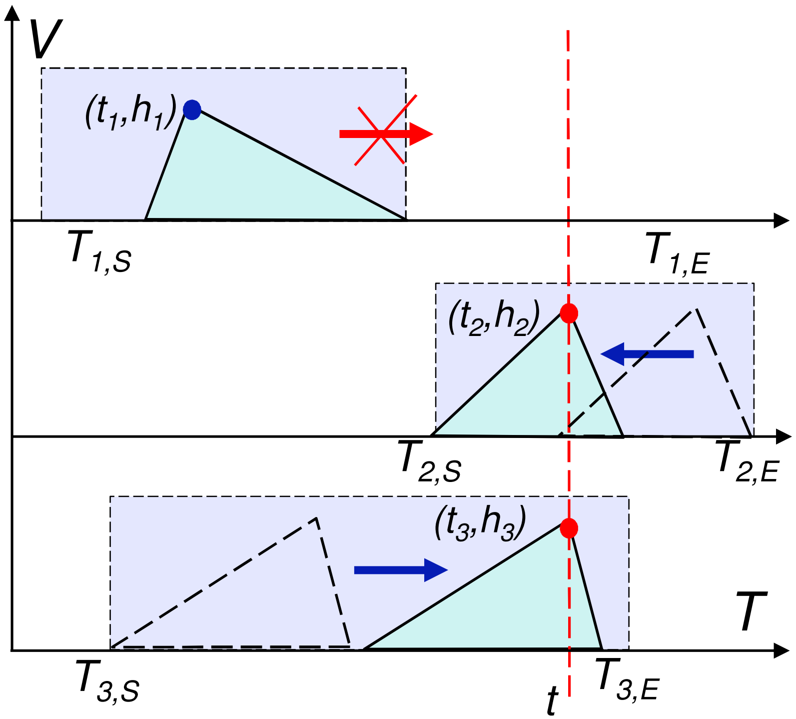
$$v_i \ge 0$$

$$T_{i,S} \le t_i \le T_{i,E}$$

Failure of LP Formulation

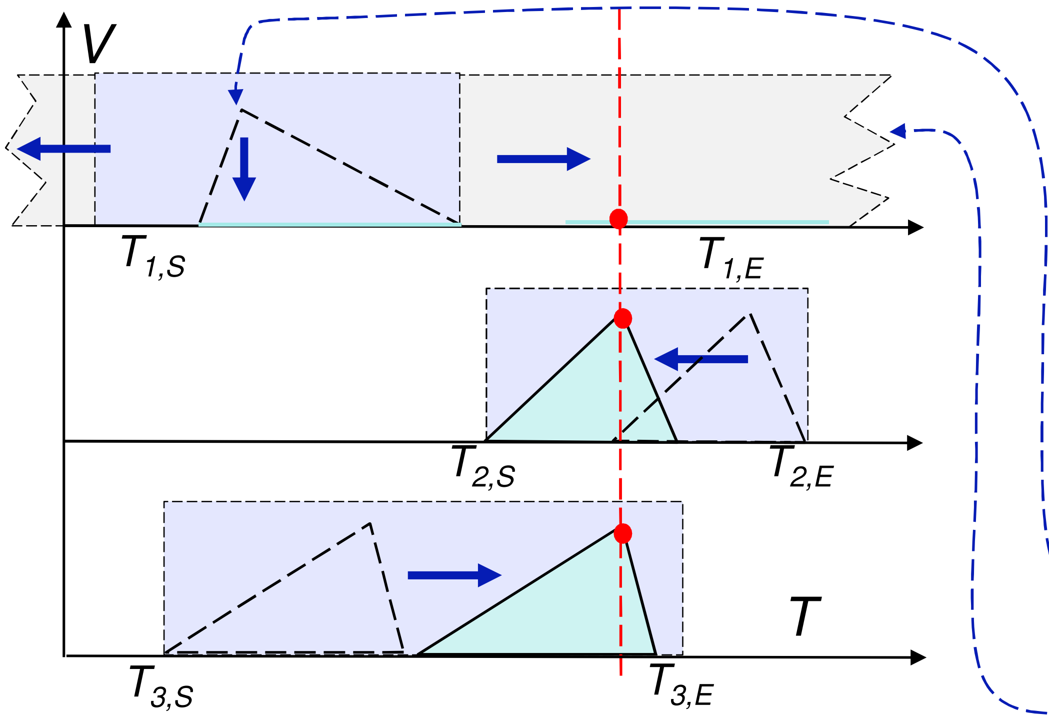
$$\underset{t,\, t_i,\, v_i}{Maximize} \quad \sum_{i=1}^{N} v_i \qquad subject\ to$$

$$(v_i - h_i) - r_i(t - t_i) \le 0$$
$$(v_i - h_i) - f_i(t - t_i) \le 0$$
$$v_i \ge 0$$
$$T_{i,S} \le t_i \le T_{i,E}$$



- **If timing windows do not intersect their constraint are not compatible**
  – Noise pulses cannot be aligned
- **LP fails to compute maximum noise value, because it is infeasible**
- **However:**
  – Worst aggressor alignment exists and maximum noise value can be computed

Deterministic Aggressor Alignment by MILP



$$\underset{t,\, t_i,\, v_i,\, p_i}{Maximize} \quad \sum_{i=1}^{N} v_i \qquad subject\ to$$

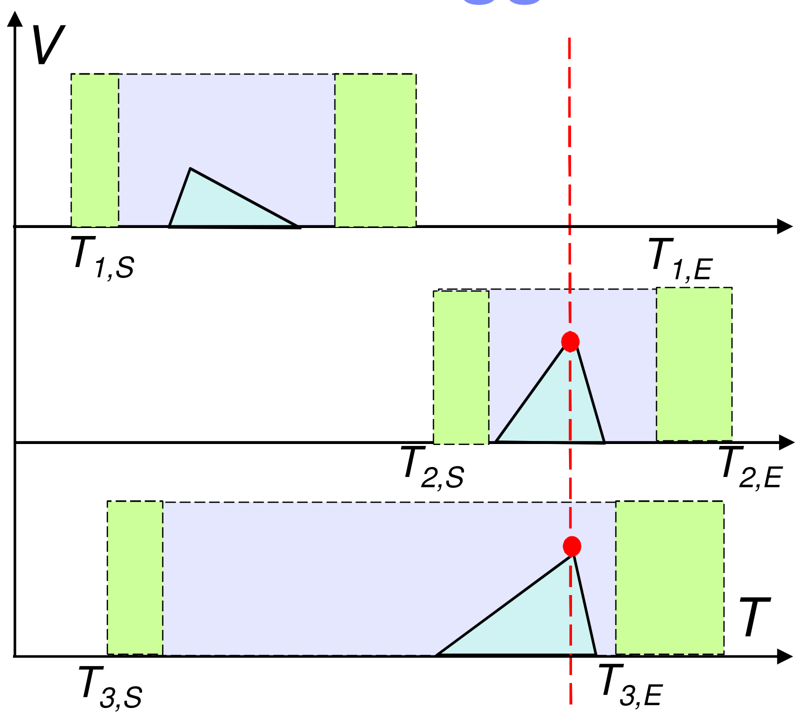
$$(v_i - p_i h_i) - r_i(t - t_i) \leq 0$$

$$(v_i - p_i h_i) - f_i(t - t_i) \leq 0$$

$$T_{i,S} - \Theta(1 - p_i) \leq t_i \leq T_{i,E} + \Theta(1 - p_i)$$

- **Introduce binary variables p_i to facilitate selection of worst aggressor set**
- **Pulse heights are multiplied with p_i**
- **Timing window constraints are modified by adding relaxation term**
- **If p_i=0**
  - *i-th* noise pulse is excluded from consideration
  - *i-th* timing window is expanded to make window constraint is always satisfied
- **If p_i=1 *i-th* noise pulse and its window are remained the same**

Variational Aggressor Alignment by MILP



- **Start and end moments of timing windows are linear functions of variational parameters**

$$T_{i,S} = T_{i,S,0} + \sum_{j=1}^{m} a_{i,S,j}\, X_j$$
$$T_{i,E} = T_{i,E,0} + \sum_{j=1}^{m} a_{i,E,j}\, X_j$$

$$\underset{t,\, t_i,\, v_i,\, X_j,\, p_i}{\textit{Maximize}} \quad \sum_{i=1}^{N} v_i \qquad \textit{subject to}$$

$$(v_i - p_i h_i) - r_i(t - t_i) \le 0$$
$$(v_i - p_i h_i) - f_i(t - t_i) \le 0$$

Variational timing constraints

$$T_{i,S,0} + \sum_{j=1}^{m} a_{i,S,j}\, X_j - \Theta(1 - p_i) \le t_i$$
$$T_{i,E,0} + \sum_{j=1}^{m} a_{i,E,j}\, X_j + \Theta(1 - p_i) \ge t_i$$

Variational parameter constraints

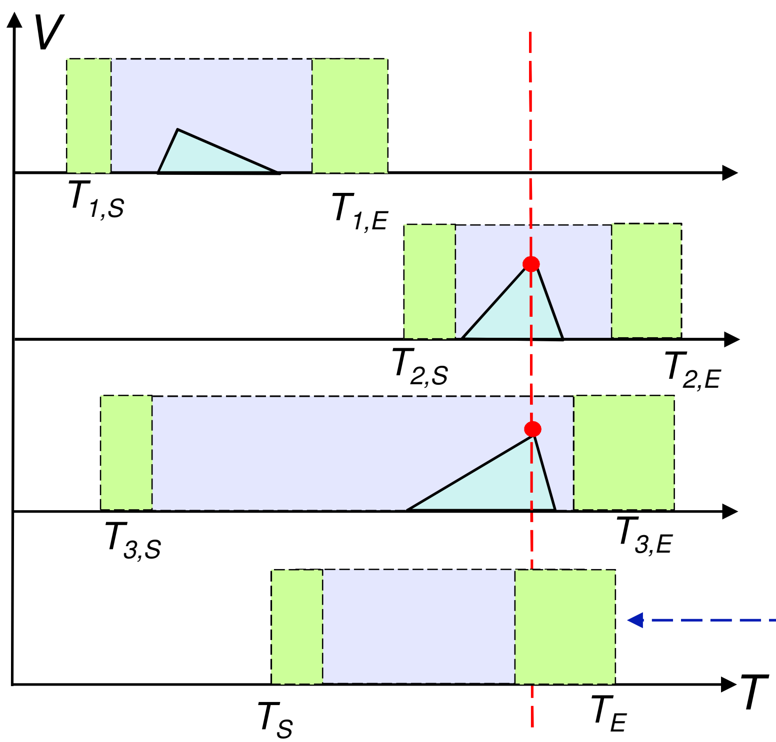
$$X_{j,min} \le X_j \le T_{j,max}$$

Computational Efficiency and Its Improvement

- **MILP is NP complete but problem of aggressor alignment is small**
  - Integer variables are always binary
  - Number of integer variables is number of aggressors (<10)
  - Only a few noise clusters require variational analysis
    - Nets without noise violations for conservatively expanded windows are not considered
    - Nets with noise violations at nominal corner are not considered
- **Matlab solves MILP for 10 aggressors in 25 iterations 30 msec in average**
- **Dimension of MILP problem can be reduced further**
  - Conservatively approximate aggressors with small noise pulses
    - Assume their perfect alignment or
    - Approximate their variability conservatively by expanding their windows
  - Conservative approximation of small sources of variations by window expansion
  - Split set of aggressors and solve MILP for each subset
  - Guide MILP procedure to analyze integer variables in optimal order
  - Add fast out in MILP when
    - Lower bound exceeds noise threshold or
    - Upper bound is lower than threshold
  - Exclude wide timing windows overlapping with other windows deterministically
  - Combine conventional linear sweeping line algorithm with MILP algorithm

Extensions: Victim Sensitivity Windows

- **Victim sensitivity window is modeled by adding its constraint to MILP formulation**



Victim sensitivity window

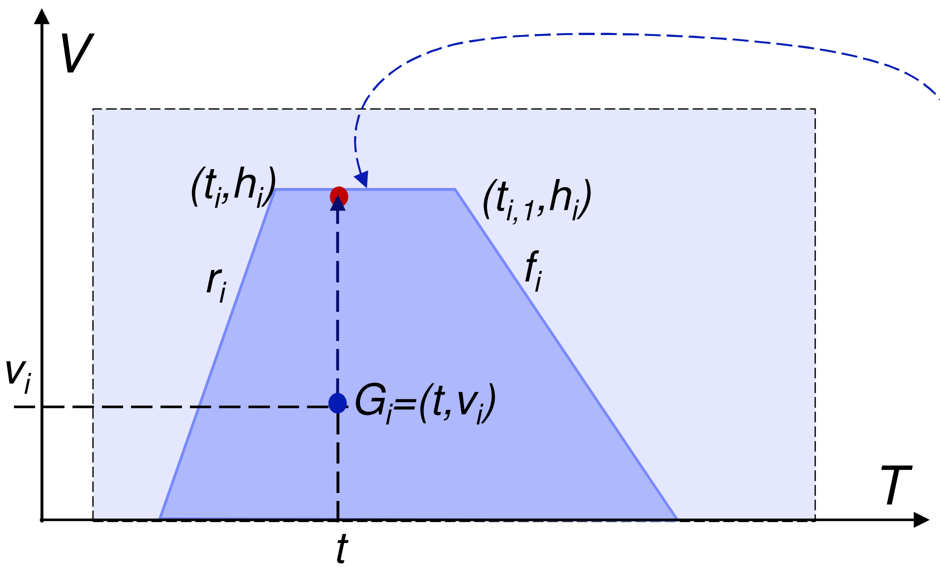
Maximize $\sum_{i=1}^{N} v_i$ *subject to*

t, t_i, v_i, X_j, p_i

$$(v_i - h_i) - r_i(t - t_i) \le 0$$
$$(v_i - h_i) - f_i(t - t_i) \le 0$$
$$T_{i,S,0} + \sum_{j=1}^{m} a_{i,S,j} X_j - \Theta(1 - p_i) \le t_i$$
$$T_{i,E,0} + \sum_{j=1}^{m} a_{i,E,j} X_j + \Theta(1 - p_i) \ge t_i$$
$$X_{j,min} \le X_j \le T_{j,max}$$
$$T_{S,0} + \sum_{j=1}^{m} a_{S,j} X_j \le t \le T_{E,0} + \sum_{j=1}^{m} a_{E,j} X_j$$

Extensions: Non-triangle Noise Pulse

- **Formulation for trapezoidal noise pulse**



Maximize v_i *subject to*

t

$$(v_i - h_i) - r_i(t - t_i) \leq 0$$

$$(v_i - h_i) - f_i\left(t - t_{i,1}\right) \leq 0$$

$$v_i \leq h_i$$

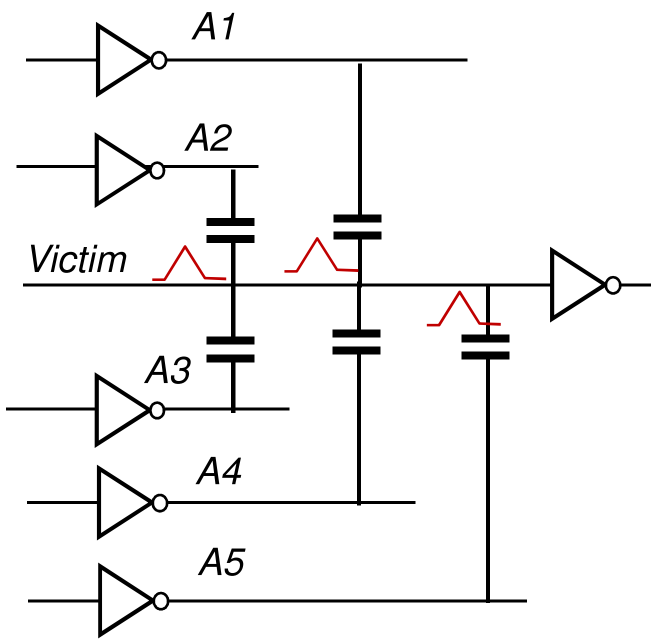
- **In general case of piece-wise linear convex pulse:**
  - Oblique segment going through point $(t_{i.j}, h_{i,j})$ with slope $s_{i,j}$ is described with constraint:

$$\left(v_i - h_{i,j}\right) - s_{i,j}\left(t - t_{i.j}\right) \leq 0$$

  - Each horizontal segment is described with constraint $v_i \leq h_{i,k}$
  - Each vertical segment is described with constraint $t \leq t_i$

Extensions: Aggressor Switching Constraints

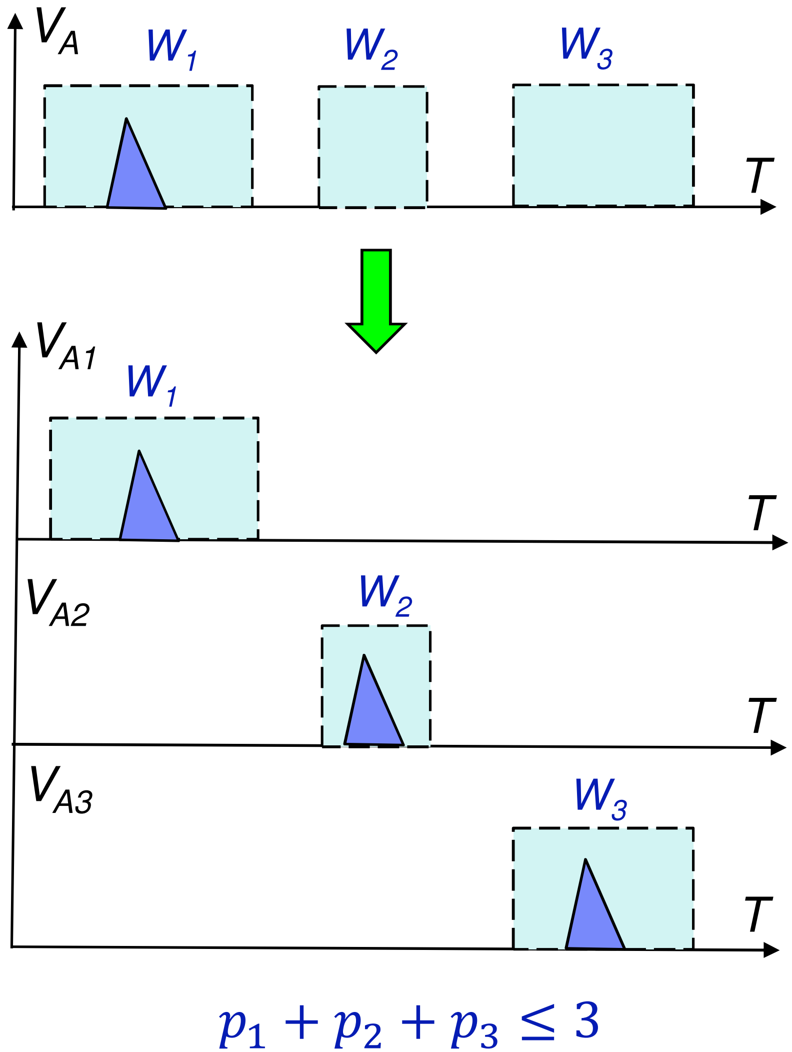
- **Circuit logic can restricts aggressors from simultaneous switching**
  - Among aggressors belonging to group *G={A1, A3, A4}* only *M* can switch simultaneously
    - If *M=1* it means mutual exclusive switching
- **MILP formulation can be extended take into account switching constraints**
  - Adding inequalities on variables controlling aggressor selection



$$\underset{t,\, t_i,\, v_i,\, X_j,\, p_i}{\text{Maximize}} \quad \sum_{i=1}^{N} v_i \qquad \text{subject to}$$

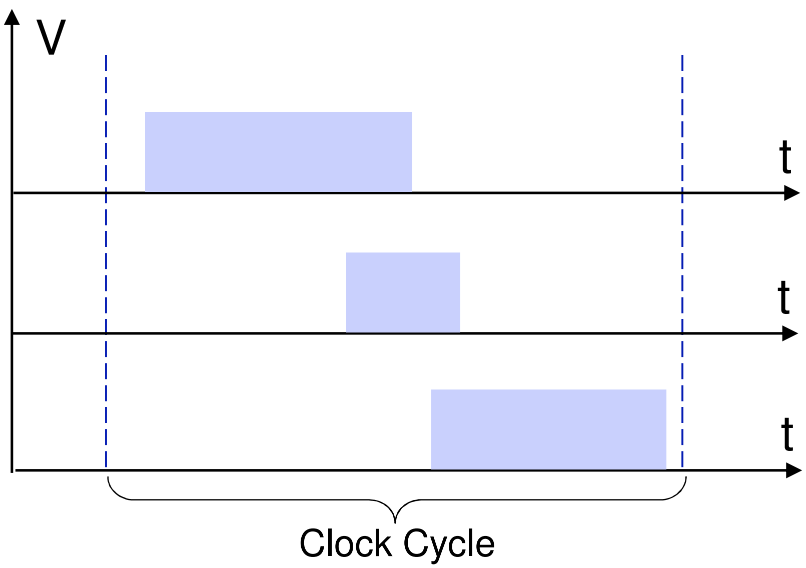
$$(v_i - h_i) - r_i(t - t_i) \le 0$$
$$(v_i - h_i) - f_i(t - t_i) \le 0$$
$$T_{i,S,0} + \sum_{j=1}^{m} a_{i,S,j} X_j - \Theta(1 - p_i) \le t_i$$
$$T_{i,E,0} + \sum_{j=1}^{m} a_{i,E,j} X_j + \Theta(1 - p_i) \ge t_i$$
$$X_{j,min} \le X_j \le T_{j,max}$$
$$\sum_{k \ni G} p_k \le M$$

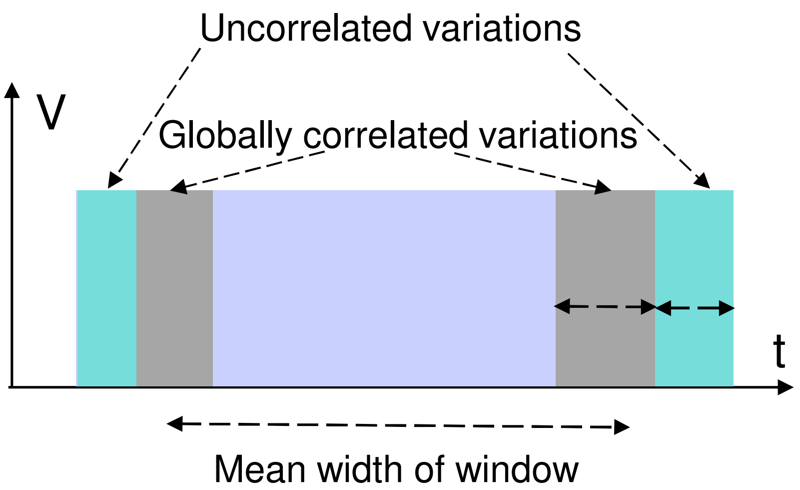
Extensions: Discontinuous Windows



- **Aggressor and victim of different clocks result in discontinuous timing windows**
- **Aggressor with timing window consisting of *M* intervals W_1, W_2, …, W_M is modeled as:**
  - M aggressors with timing windows W_1, W_2, …, W_M and
  - Constraint $\sum_{k=1}^{M} p_k \leq 1$ imposed on binary variables p_1, p_2, …, p_M of MILP formulation
    - Selects single noise pulse

Experiments with Many Aggressors: Setup





- **Modeling realistic distribution of timing windows**
  - Random distribution of timing windows inside clock cycle
  - Uniform distribution of mean values of window width
  - Random amount of correlated and uncorrelated variability
- **Same noise pulses of unit height**
  - All aggressors have same importance
- **Cases with:**
  - 3, 5 and 10 aggressors
  - Different clock cycles,
  - Different min/max values of window width
  - Different values of correlated and uncorrelated variability
- **Up to 11 sources of variations**

Experiments with Many Aggressors: Results

- **Experiment for 100 different values from nominal and conservative bounding methods**
  - Only cases requiring variational analysis
- **Best accuracy from corner enumeration. Worst accuracy from nominal analysis.**
- **Number of MILP iterations much fewer than number of corners**

Exp Num	# Agg	# MILP iter	Error of noise computation								
			Nominal			Bounding			Enumerating		
			#Err	Max	Avr	#Err	Max	Avr	#Err	Max	Ave
1	3	3.27	79	-2	0.69	31	1.0	0.22	1	-0.19	0.002
2	3	1.48	69	-1	0.58	43	1.0	0.33	3	-0.37	0.01
3	3	1.62	75	-1	0.52	43	1.0	0.28	2	-0.41	0.005
4	5	11.6	94	-2	0.75	31	1.0	0.23	7	-0.66	0.024
5	5	4.1	85	-2	0.71	36	1.0	0.27	3	-0.5	0.012
6	5	3.94	83	-1.7	0.67	42	1.73	0.31	2	-0.1	0.001
7	10	56.3	91	-4	1.27	62	2.0	0.52	21	-1.17	0.12
8	10	16.4	97	-4	1.29	62	2.0	0.58	23	-1	0.098
9	19	17.1	90	-2.4	0.93	58	1.94	0.45	7	-.94	0.03

Conclusions

- **Analyzed cross-talk aggressor alignment under process variation**
- **Showed that even enumeration of all corners fails to find worst alignment**
- **Developed MILP technique for computing worst aggressor alignment under process variation**
  - Conservative non-statistical approach compatible with conventional corner-analysis
- **Extended MILP technique for**
  - victim sensitivity windows,
  - non-triangle noise pulses,
  - aggressor switching constraints,
  - discontinuous timing windows
- **Many special problems of cross-talk aggressor alignment can be solved with same MILP solver**
- **MILP solver computes not only worst noise and alignment, but also worst corner and sensitivities of noise to variations**
- **Experiments showed that MILP of aggressor alignment can be solved efficiently**
- **Outlined several methods (exact and heuristic) for further improving computational efficiency**