Importance of Modeling Non-Gaussianities in STA in sub-16nm Nodes

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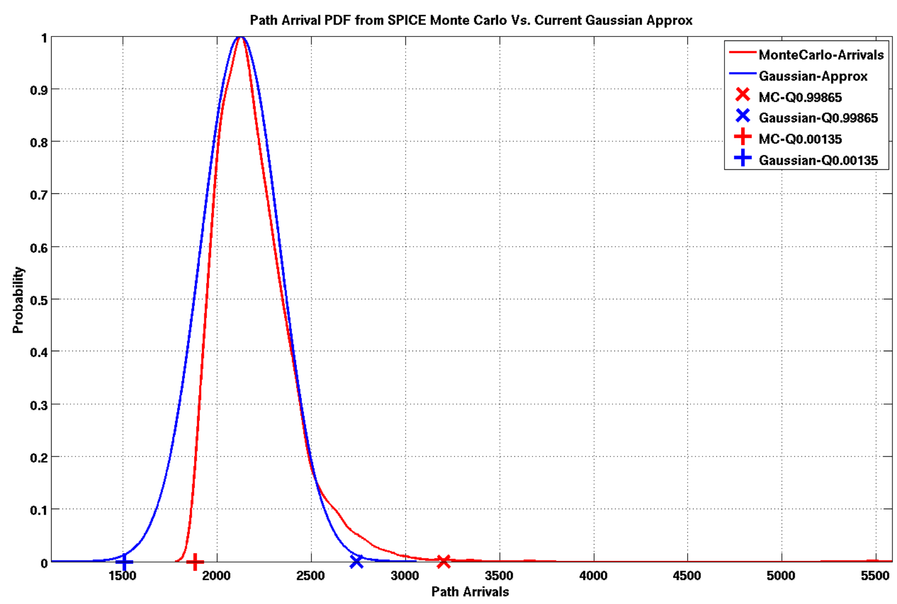
Introduction

- SSTA is very accurate, but not widely adopted
  - Designers use corners instead of inter-die random variables
  - SSTA run-time with mismatch 3-10X over a single corner run
  - Too expensive in implementation tools with MMMC
  - Variation-aware CCS/ECSM characterization very expensive
- AOCV (fast but inaccurate), SOCV/LVF (good tradeoff between performance and accuracy)
- SOCV/LVF model delay/slew/constraint sigma as a NLDM table over the slew-load ranges

Problem Description

- In sub-16nm and even 28nm at low VDDs (< 0.6V)
  - Delay distributions significantly non-Gaussian
  - Low voltage (VDD-Vth) headroom
- LVF with sigma tables not accurate enough to model quantiles



Qtile(0.99865) != (μ+3σ)
Qtile(0.00135) != (μ-3σ)

Problem Description

- Need to model delay/slew/constraints with Non-Gaussian distributions
- Propagate distributions through timing graph/paths with minimal runtime impact
- Propose UDG extensions to LVF to include first 3 moments
  - Mean-shift ***(Δμ)*** , Std-dev ***(σ)*** , Skewness ***(γ)***
  - Accurately model arrival/slack distribution of STA paths
  - Keep run-time impact low compared to run with only sigma tables

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Current LVF Model

```
• ocv_sigma_cell_rise (dly_temp_3x3) {
      sigma_type : early;
      index_1 (0.0126, 0.0316, 0.0794");
      index_2 ("0.00199, 0.0049, 0.0124");
      values ( \
        "0.0716354, 0.154961, 0.363279", \
       "0.07307970, 0.156313, 0.364406", \
       "0.07670830, 0.159710, 0.367235", \   ); }

• ocv_sigma_cell_rise (dly_temp_3x3) {
      sigma_type : late;
      index_1 (0.0126, 0.0316, 0.0794");
      index_2 ("0.00199, 0.0049, 0.0124");
      values ( \
            "0.151828, 0.330933, 0.778703", \
            "0.152527, 0.331790, 0.779956", \
            "0.154283, 0.333941, 0.783103", \  ); }
```


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Extend LVF with UDG

```
ocv_mean_shift_cell_rise (dly_temp_3x3) {
    index_1 (0.0126, 0.0316, 0.0794");
    index_2 ("0.00199, 0.0049, 0.0124");
    values ( \
     0.0125328, 0.0274474, 0.064734, \
     0.0124352, 0.0273842, 0.0647568 \
     0.0121902, 0.0272255, 0.0648139 \ ); }
```

$$\mu = E(D(x)) = \int_{-\infty}^{+\infty} D(x) pdf(x) dx$$

$$\sigma = \sqrt{E((D(x)-\mu)^2)}$$

$$\gamma = \frac{\sqrt[3]{E((D(x)-\mu)^3)}}{\sigma}$$

```
ocv_std_dev_cell_rise (dly_temp_3x3) {
    index_1 (0.0126, 0.0316, 0.0794");
    index_2 ("0.00199, 0.0049, 0.0124");
    values ( \
        0.0576022, 0.125316, 0.294603, \
        0.0580914, 0.125831, 0.295186, \
        0.0593250, 0.127126, 0.296648, \
   ); }
```

```
ocv_skewness_cell_rise (dly_temp_3x3) {
    index_1 (0.0126, 0.0316, 0.0794");
    index_2 ("0.00199, 0.0049, 0.0124");
    values ( \
        0.978999, 0.982136, 0.983660, \
        0.973143, 0.979837, 0.983088, \
        0.958501, 0.974089, 0.981658, \
   ); }
```

- Other constructs - ***ocv_mean_shift_rise_transition, ocv_mean_shift_rise_constraint***

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Core Statistical Operations

- Statistical max operation
  - $\boldsymbol{A_{max}(\mu_{max},\sigma_{max},\gamma_{max}) = max\ (A_j(\mu_j,\sigma_j,\gamma_j), A_k(\mu_k,\sigma_k,\gamma_k)\ )}$
  - Assume PDF for $A_j(\mu_j,\sigma_j,\gamma_j)$, e.g., *log-normal, skew-normal, beta, cauchy, chi-squared, gamma, etc.*
  - Compute $\boldsymbol{(\mu_{max},\sigma_{max},\gamma_{max})}$ based on moment matching & numerical integration, while bounding ***correlation(A_j, A_k)***
- Statistical add operation
  - $\boldsymbol{A_{i+1}(\mu_{i+1},\sigma_{i+1},\gamma_{i+1}) = A_i(\mu_i,\sigma_i,\gamma_i) + D_{\{i,i+1\}}(\mu_d,\sigma_d,\gamma_d)}$
  - Assume a PDF for $A_j(\mu_j,\sigma_j,\gamma_j)$
  - Compute $\boldsymbol{(\mu_{i+1},\sigma_{i+1},\gamma_{i+1})}$ considering ***correlation(A_j, $D_{\{i,i+1\}}$)***

Arrival/Slack Quantile Computation

- Given ***(μ, σ, γ)*** of arrivals/slack times, compute quantiles 0.99865 and 0.00135
- To model Quantiles: *log-normal, skew-normal, beta, cauchy, chi-squared, gamma, etc.*
- PBA only does add, quantiles obtained with good accuracy compared to MC SPICE
- GBA does max & add, results accurate enough to bound PBA

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Results Setup

- Nodes are 10nm, 28nm at ultra-low-VDD (< 0.6v)
- Cell library char using Variety for ***(Δμ, σ, γ)***
- About 1000 paths in path-based-analysis (PBA) mode in Tempus
  - (I) With ***Δμ and σ,*** (II) With ***Δμ, σ, and γ***
- Compare with 5000 Monte Carlo SPICE simulations
- Quantiles 0.99865, 0.00135 compared to MC SPICE

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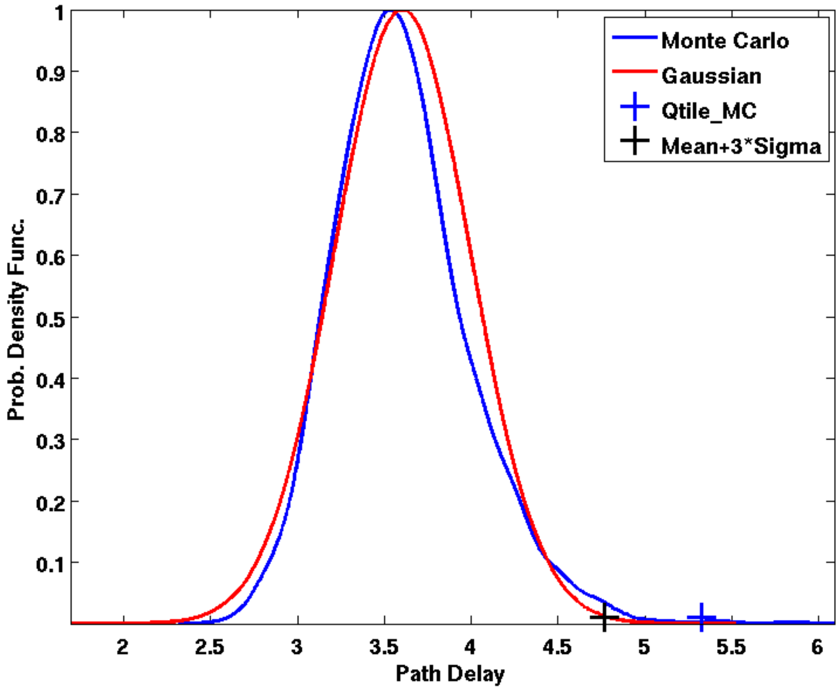
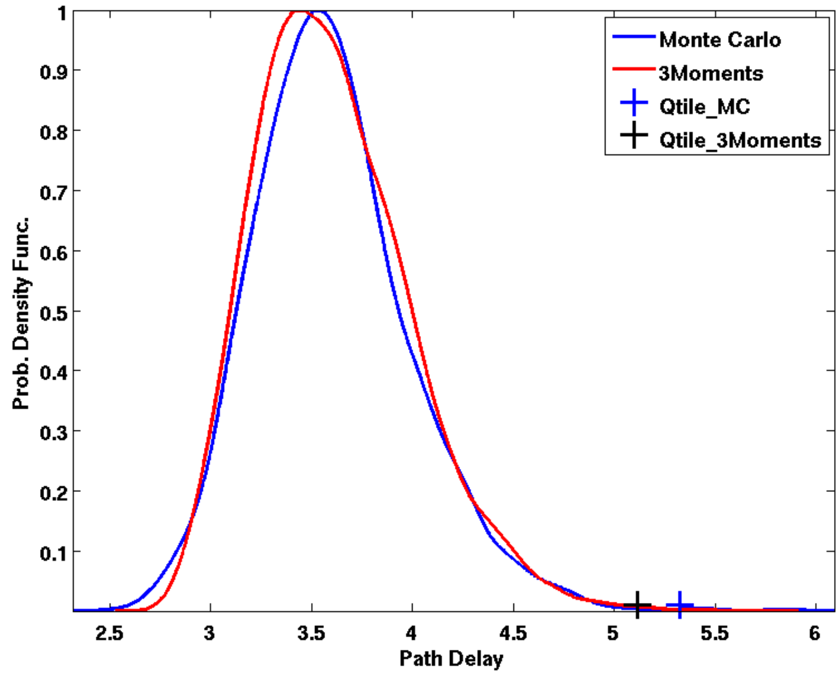
Q0.99865 of Arrivals on 20 Paths

Path #	Qtile_MC nanosec	Qtile_3Moments		Qtile_Gaussian	
		nanosec	% Err2MC	nanosec	%Err2MC
Path 1	9.385	8.569	-8.7	7.566	-19.4
Path 2	12.829	12.17	-5.1	10.498	-18.2
Path 3	8.209	7.551	-8	6.92	-15.7
Path 4	9.569	8.954	-6.4	8.188	-14.4
Path 5	11.243	10.997	-2.2	9.691	-13.8
Path 6	8.505	8.292	-2.5	7.34	-13.7
Path 7	11.071	10.761	-2.8	9.603	-13.3
Path 8	8.563	8.126	-5.1	7.436	-13.2
Path 9	6.434	6.197	-3.7	5.654	-12.1
Path 10	9.551	9.198	-3.7	8.449	-11.5
Path 11	7.361	7.232	-1.8	6.562	-10.8
Path 12	10.083	9.715	-3.7	8.99	-10.8
Path 13	5.324	5.099	-4.2	4.764	-10.5
Path 14	7.833	7.907	0.9	7.082	-9.6
Path 15	8.089	7.978	-1.4	7.32	-9.5
Path 16	4.693	4.522	-3.6	4.249	-9.5
Path 17	6.296	6.087	-3.3	5.705	-9.4
Path 18	7.923	7.762	-2	7.202	-9.1
Path 19	7.252	7.18	-1	6.609	-8.9

PDF Plots for a Sample Path



PDF from using Δμ and σ



PDF from using Δμ, σ and γ

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Conclusions

- Significant optimism/yield-risk in assuming Gaussian delay PDF at ultra-low VDDs at 28nm and below
- Mean-shift & skewness modeling essential at ultra-low VDDs
- Char tools exist to char ***(Δμ, σ, γ)*** accurately at no extra cost
  - ***(Δμ, σ, γ)*** added as UDGs to existing LVF libs
- Our PBA results show good accuracy in modeling the non-Gaussian effects on delay quantiles
- Run-time impact of skewness modeling is small (about 5-10%)

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Questions

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