



# Practical Statistical Static Timing Analysis with Current Source Models

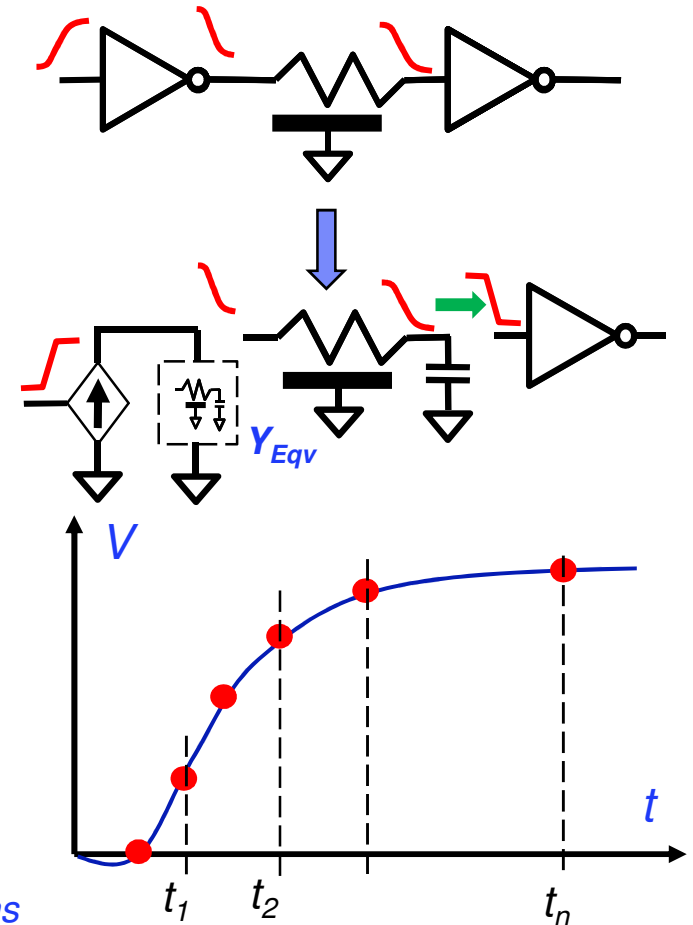
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## Driver and Signal Modeling

- **Delay/Slew/Ceff driver model is too inaccurate**
- **Current Source Model (CSM) has better accuracy**
  - Einstimer uses efficient implementation based on dynamic effective capacitance
    - *P Feldmann, S Abbaspour, D Sinha, et al., Driver waveform computation for timing analysis with multiple voltage threshold driver models, DAC 2008*
- **Waveforms model is necessary companion**
  - Better accuracy but higher computational cost and memory footprint
- **Einstimer implements statistical timing**
  - Delays, slews, ATs, RATs, etc. are modeled with linear or bilinear forms
    - Cross-terms model interaction of sources of variations like dependence of variability on Vdd



Linear terms

Cross-terms

$$t = t_0 + t_V \Delta V + t_1 \Delta X_1 + \dots + t_R \Delta R_a + t_{V,1} \Delta V \Delta X_1 + \dots + t_{V,R} \Delta V \Delta R_a$$

**Efficient statistical model of waveform is needed**

## Statistical waveform models

- Time points are modeled with statistical canonical form

- H.Fatemi, S.Nazarian, and M.Pedram, "Statistical logic cell delay analysis using a current-based model, DAC2006"*

- High memory consumption and redundancy

- Time moments are highly correlated

- Statistical waveform model approximates variability with timing shift and scale

$$V(t) = V_{nom}(t - S[t - t_0] - D)$$

- Nominal waveform  $V_{nom}(t)$  defines shape

- Canonical form of shift acts as statistical delay

$$D = d_0 + s_V \Delta V + d_1 \Delta X_1 + \dots + d_R \Delta R_a$$

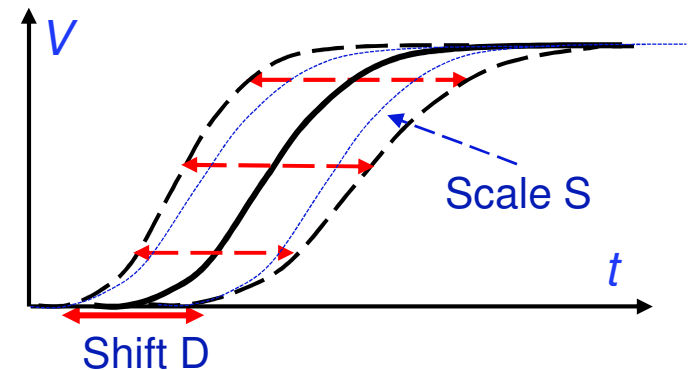
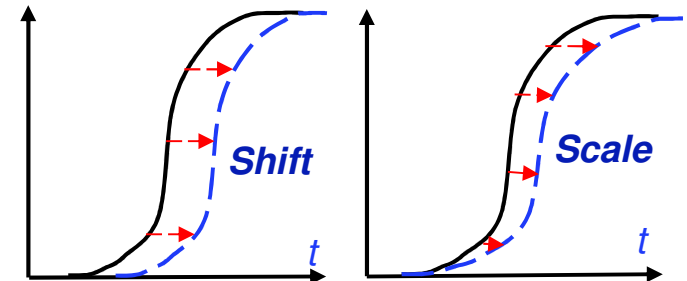
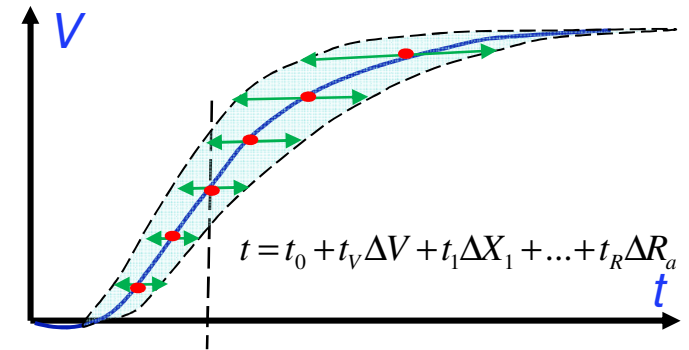
- Canonical form of scale acts as statistical slew

$$S = s_0 + s_V \Delta V + s_1 \Delta X_1 + \dots + s_R \Delta R_a$$

- *V. Zolotov, J. Xiong, C. Visweswariah, S. Abbaspour, D. Hathaway, "Compact modeling of variational waveforms", ICCAD 2007 computes sensitivities by solving CSM differential equations, which is rather inefficient.*

- Efficient technique for computing and propagating statistical waveform is needed

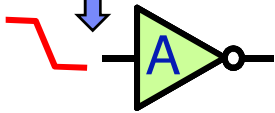
- Support incremental timing



# Statistical waveform propagation (“Partial CSM”)

Statistical AT & slew at gate input

$$S = s_0 + s_V \Delta V + \dots + s_R \Delta R_a$$



Compute ATs and slews for base corners

Solve CSM for all base corners to get waveforms at gate output

Compute and store nominal waveform, statistical AT and slew

Construct corner waveforms at interconnect input by **shift and scale** to match corner ATs and slews

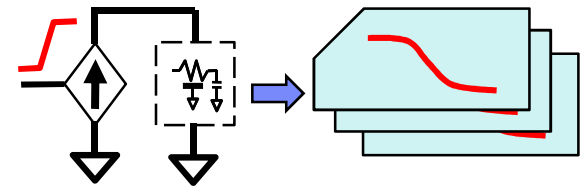
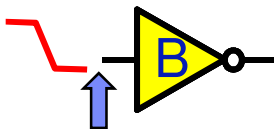
Construct interconnect models (ROMs) for all base corners

Propagate waveforms for all base corners through interconnect

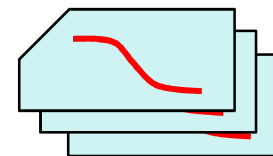
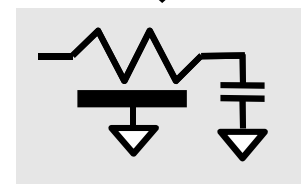
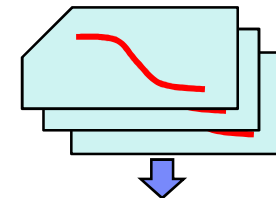
Compute and store statistical AT and slew

Statistical AT & slew at next gate input

$$S = s_0 + s_V \Delta V + \dots + s_R \Delta R_a$$



$$V_{\text{nom}}(t) \quad s_0 + s_V \Delta V + \dots + s_R \Delta R_a$$



## Study of statistical timing models

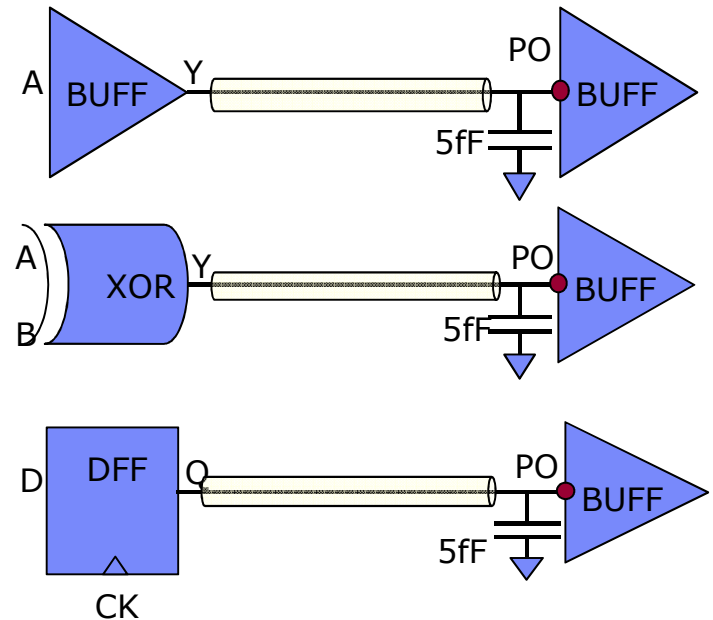
### ■ 4 methods of statistical timing:

- “No CSM” – NLDM instead of CSM
- “Full CSM” - waveforms for all base corners
  - Base corners waveform are needed for incremental timing
- “Partial CSM” –nominal waveform with statistical shift&scale
  - Waveforms at other corners are constructed by matching delays and slews of conventional statistical timing
- “Ramp CSM” – no waveforms stored
  - Gate is modeled with CSM to get delay & slew
  - Linear ramp is propagated through interconnect

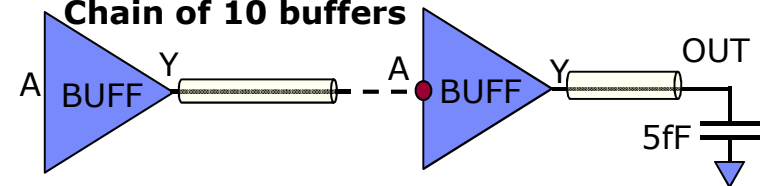
### ■ 8 PVT corners;

- FF Vdd = 1.1/0.6 T= 85C
- FF Vdd = 1.1/0.6 T= -10C
- SS Vdd = 1.1/0.6 T= 85C
- SS Vdd = 1.1/0.6 T= -10C
  - Required for PT cross-term

### Four 14nm test circuits

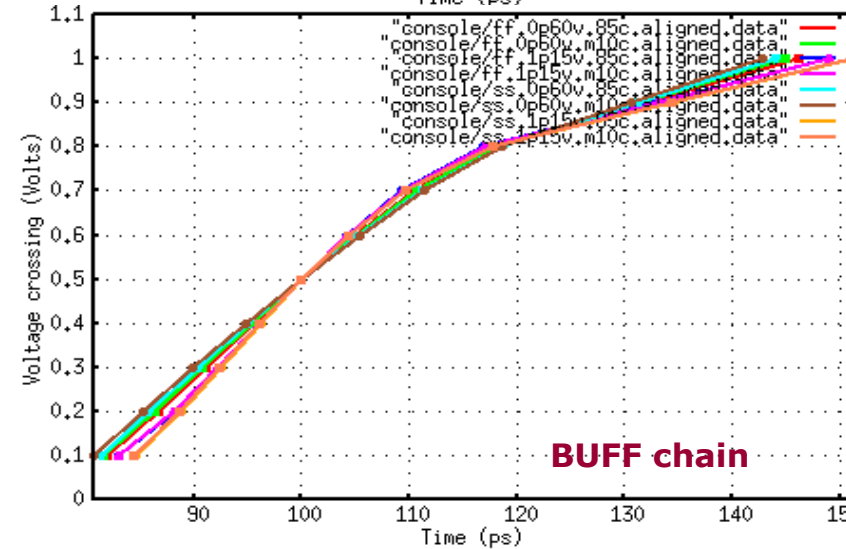
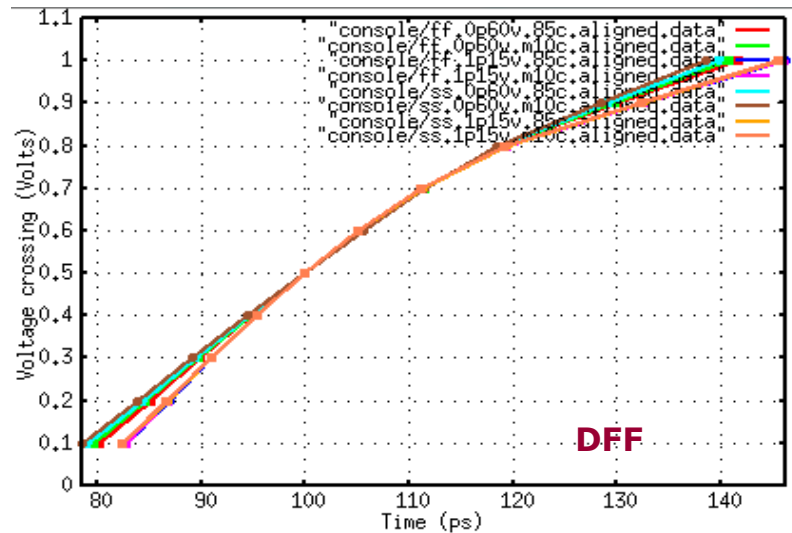
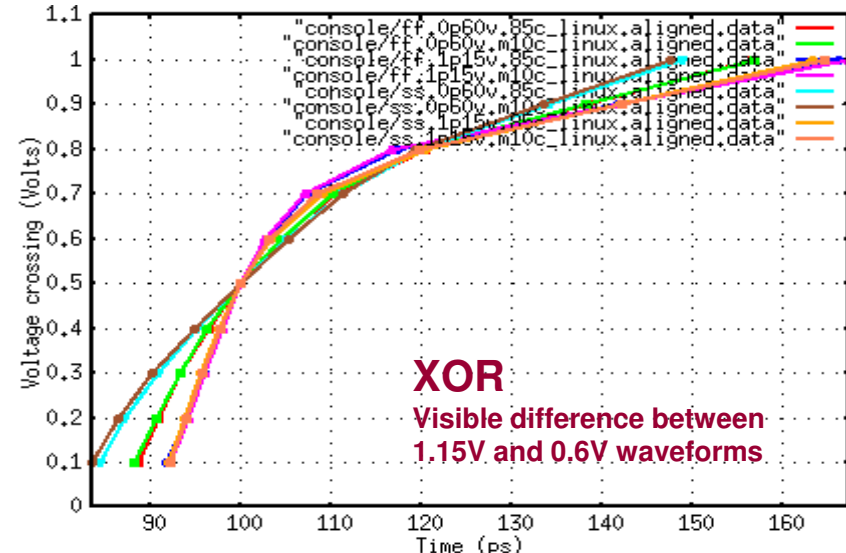
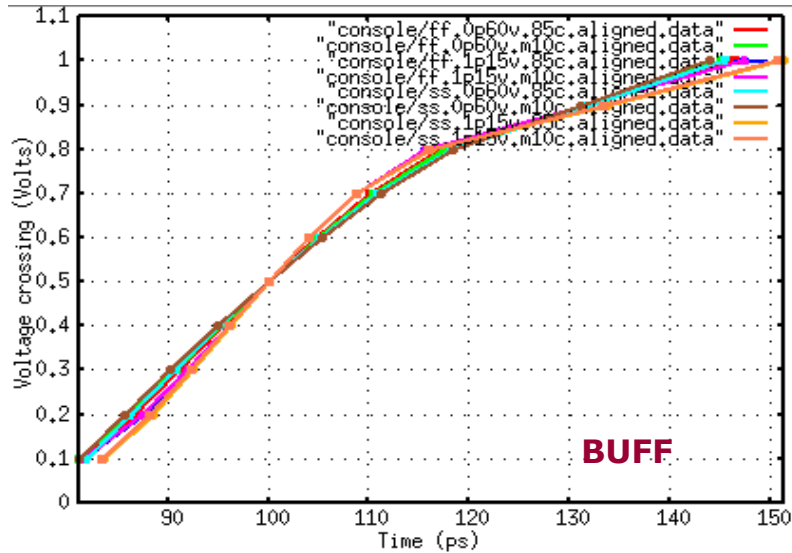


### Chain of 10 buffers



Resistive wire: ~30-100 fF Ctotal, ~20-40 fF Ceff, 0.3-1 kOhm resistance]

# Aligned corner waveforms



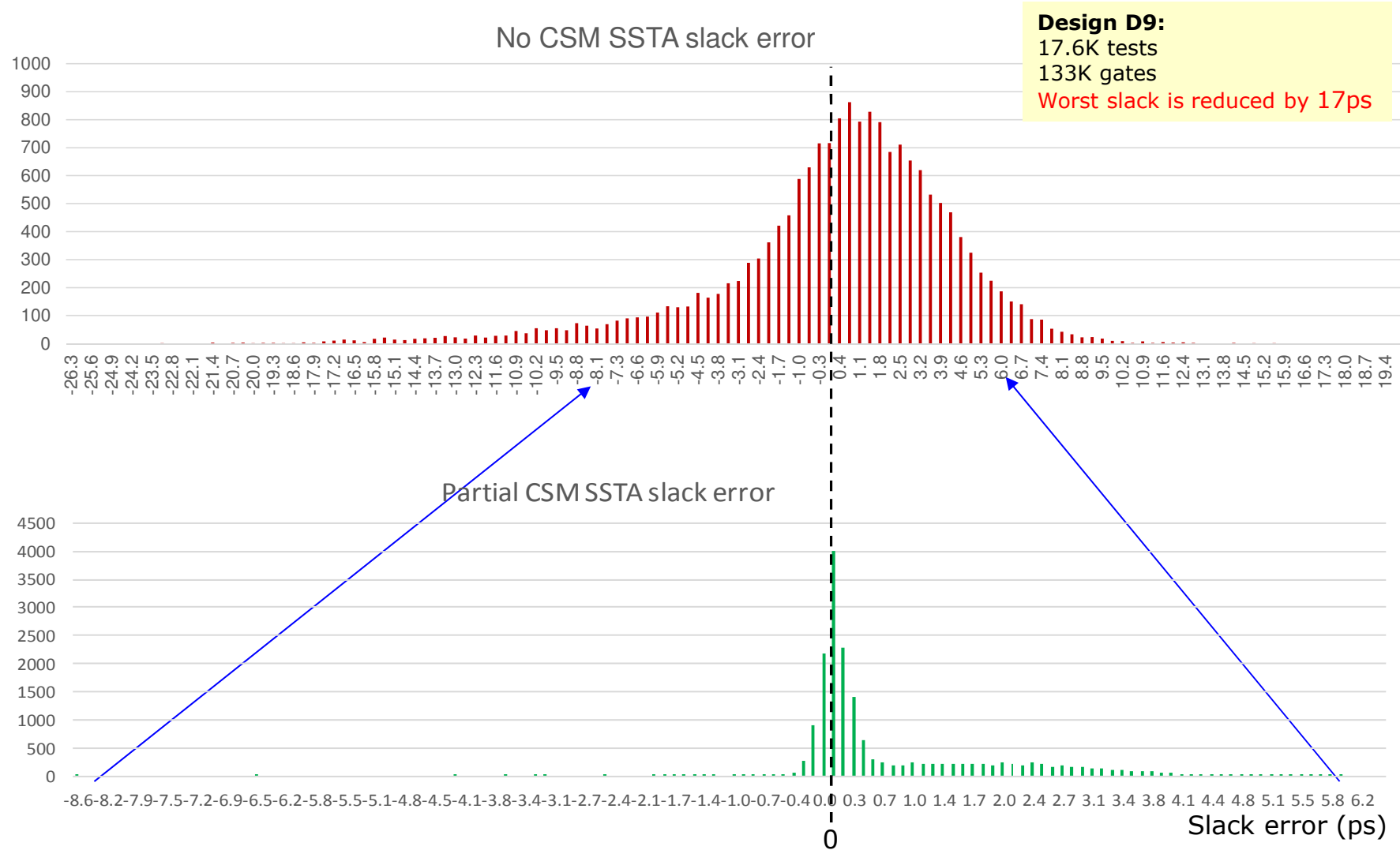


## Base corner selection

Corner P/V/T	Max error of delay/slew (%)				Average error of delay/slew (%)			
	BUFF	XOR	DFF	Chain	BUFF	XOR	DFF	Chain
FF/1.1/-10	4.1/0.6	4.7/4.1	8.1/2.9	2.0/0.27	-1.9/-0.2	-1.4/2.1	1.7/1.5	-0.85/-0.06
FF/1.1/+85	4.4/0.7	3.8/3.8	9.0/2.6	2.0/0.26	-2.1/-0.2	-0.5/1.7	2.3/1.0	-0.84/-0.06
SS/1.1/-10	2.3/0.5	7.5/3.5	12.6/2.6	1.0/0.16	-0.2/0	2.4/1.2	4.7/0.8	-0.04/-0.03
SS/1.1/+85	2.1/0.5	9.1/3.2	13.2/2.4	1.0/0.15	0/0	3.6/0.8	5.0/0.6	-0.04/-0.02
FF/0.6/+85	1.3/0.4	7.7/1.9	7.0/1.7	0.9/0.15	-0.2/0.1	2.1/-0.2	-0.3/-0.3	-0.01/-0.03
FF/0.6/-10	1.0/0.6	6.2/2.0	6.7/1.6	0.9/0.18	-0.2/0.2	0.9/-0.1	-0.6/-0.4	-0.01/-0.04
SS/0.6/+85	1.3/0.8	6.3/1.9	9.9/3.0	1.5/0.05	0.65/0.5	-3.0/-1.2	-5.6/-1.7	0.72/0.02
SS/0.6/-10	0.8/0.9	7.5/1.9	10.7/3.2	1.4/0.05	0.13/0.5	-4.3/-1.3	-6.5/-2.0	-0.59/0

- Accuracy highly depends on selection of base (nominal) corner
  - No corner with best accuracy for all circuits both for delays and slews
- Selected corner with best tradeoff accuracy

# Hold slack error histogram





## Timing accuracy, run time, memory footprint

Design	Num gates	Num tests	No CSM AT/Slew Error (%)	CSM AT/Slew Error (%)	Slack Accur Improvement vs No CSM	Run time vs No CSM	Memory vs no CSM
D6	17K	2.1K	1.3/12.1	0.2/0.0	6.9%	16%	3.6%
D7	23K	3.5K	0.1/1.9	0.0/0.2	4.2%	16%	4.6%
D8	30K	6.6K	2.3/13.2	0.5/0.0	5.4%	4%	4.1%
D9	133K	17.6K	1.6/13.2	0.3/0.0	7.1%	15%	15%
D10	170K	34.1K	1.9/13.3	0.3/0.0	1.4%	15%	12.5%
D11	1.63M	196K	1.6/12	0.3/0.1	5.2%	16%	17.5%

- Typical “No CSM” error in AT 1.3-1.9% (worst 18ps) , in Slew 12-13% (worst 17ps)
- Typical “Partial CSM” error in AT 0.3%, in slew 0.0-0.1%
- Up to 7% slack improvement (17ps) for representative 250ps clock cycle
- Reasonable run time overheads 16%
- Additional memory 1-1.5GB per 1M gates

## Conclusions

- **Studied implementations of CSM waveforms in statistical timing for 14nm high performance designs**
  - Naïve approach results in manifold memory overhead
- **Developed method of propagating statistical waveforms**
  - Statistical shift and scale technique was modified for better performance and lower memory footprint.
- **Achieved substantial slack accuracy improvement**
  - Up to 17ps, i.e. 7 of 250ps cycle time
- **Run time increase does not exceed 16%**
- **Memory overhead is about 1.1GB per 1M gates**
  - Does not exceed 17.5%
- **Further accuracy improvement can be achieved**
  - Dynamic selection/construction of nominal corner for shift and scale representation