Statistical Path Tracing in Timing Graphs

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Background

- Path tracing is a key step in static timing analysis and optimization
  - Given a (set of) timing end-point(s) in a graph, trace backward and find a set of most critical paths leading to each end-point
  - Applications
    - Reporting (top N critical paths)
    - Common path pessimism reduction (CPPR)
    - Optimization of a set of most critical paths
- In a deterministic timing environment, backward path tracing is performed using notion of critical arrival time (AT)
  - In this case, path tracing using AT is equivalent to tracing using slack
- In a statistical timing environment, backward path tracing is not a simple extension of path tracing during deterministic timing
- Possible approaches to path tracing
  - **DPT**: (**D**eterministic **P**ath **T**racing) Based on deterministic AT or slack – **Incorrect** (example follows)
  - Based on statistical AT (projected, tightness-probability based, etc.) – **Incorrect** (example follows)
- Statistical required arrival time (RAT) has variability, plays a role in which path(s) are most critical (and at what corner)
- **SPT: S**tatistical (slack based) **P**ath **T**racing is the correct way
  - However, how to rank order paths? Criticality – Definition?

Linear Canonical Form (LCF)

* courtesy Chandu Visweswariah (IBM Research)

$$a_0 + a_1 \Delta X_1 + a_2 \Delta X_2 + \cdots + a_N \Delta X_N + a_R \Delta R$$

Mean (nominal value)

Sensitivities

<u>Deviation</u> of global sources of variation from their nominal values

Random uncertainty (deviation from nominal value)

- What does this canonical form have to do with statistical timing?
  - determining the a_1, a_2, $\cdots$, a_N and a_R coefficients is sensitivity analysis
  - but if you think of ΔX_1, ΔX_2, $\cdots$, ΔX_N, ΔR as random Gaussian variables, then this result is a statistical result (or linear regression) which can be plotted as a distribution

LCF Math Operations

- Stat Add/Subtract
  - Mean and regular sensitivities are simply added/subtracted
  - Random sensitivity is always RSSed (root sum squared)
- Stat Min/Max
  - Clark's linear Gaussian approximation using tightness probabilities is used
  - Often introduces additional random term
- Projection: Get a number out of a Distribution (LCF)
  - Given corner $c = \{c_1, c_2, \ldots, c_N, c_R\}$
  - Define $Proj_c(A) = a_0 + \sum_{i=1}^{N} a_i c_i + \ a_R c_R$
  - Often $c_i = \{-3, +3\}$. Also, sensitivities are usually RSSed to reduce pessimism.

No guarantee of continuity in projected slack along a path

$$Proj(MIN(A,B) \neq MIN(Proj(A), Proj(B))$$
$$Proj(MAX(A,B) \neq MAX(Proj(A), Proj(B))$$

- For simplicity, we will assume only simple LCFs in this paper.
  - All Timing quantities such as Delay, Arrival Time (AT), Slew, Required Arrival Time (RAT), and Slack will be assumed to be simple LCFs.

$$Slack = \begin{Bmatrix} RAT - AT \ \ if\ Late\ \ Mode \\ AT - RAT \ \ if\ Early\ Mode \end{Bmatrix}$$

Stat Math

Late Mode Test Example

- Late Mode Stat Arrival Times:

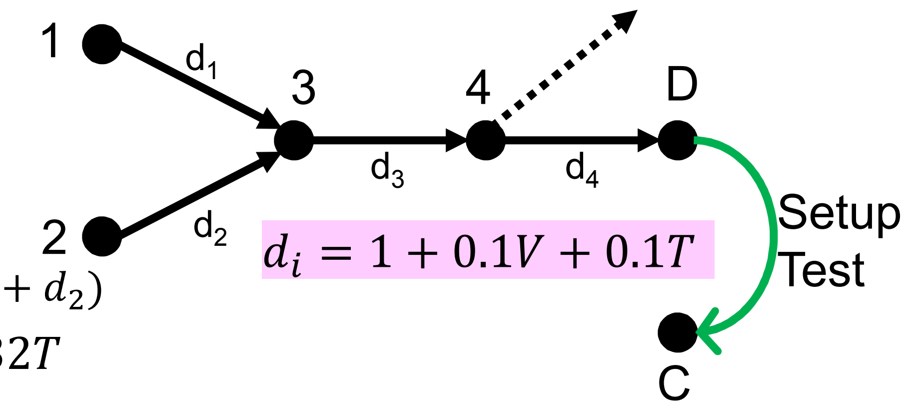
$AT_1 = \ 99 + 3.9V + 2.9T$

$AT_2 = 110 + 0.3V + 0.2T$

Dropping Δ in notation

$AT_1 + d_1 = 100 + 4V + 3T$

$AT_2 + d_2 = 111 + 0.4V + 0.3T$



$d_i = 1 + 0.1V + 0.1T$

- Stat MAX at 3: $AT_3 = MAX(AT_1 + d_1, AT_2 + d_2)$

$AT_3 = 111.011 + 0.167R + 0.426V + 0.32T$

  - Tightness Probabilities: $tp_1 = 0.007254, tp_2 = 0.992746$
    - Closer to $AT_2 + d_2$. So, based on AT (Det. or Stat.), Node 2 is more “critical” than 1 feeding 3.
  - Random appears in MAX even though incoming ATs had no random

- RAT on D: $RAT_D = 112.523 + 0.6V + 0.5T$

- Back-propagate RAT from D to 3 (fanout at 4 does not contaminate RATs at either 4 or 3):

$RAT_D - d_4 - d_3 = RAT_3 = 110.523 + 0.4V + 0.3T$

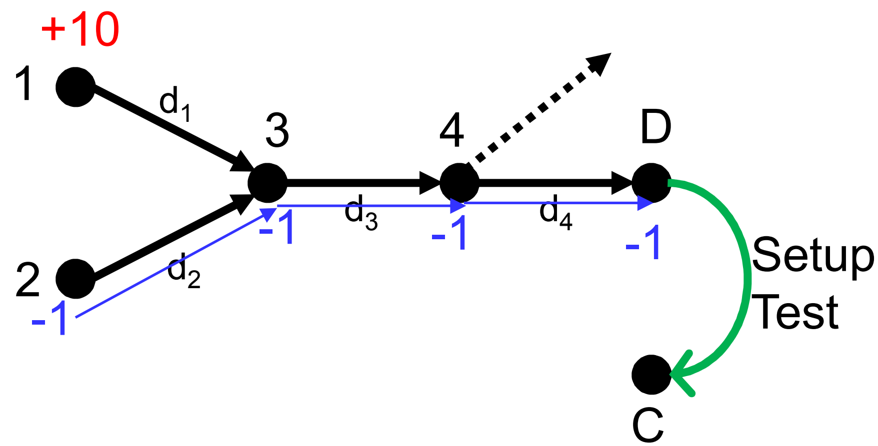
- Slack = RAT – AT (random is RSSed; hence positive):

$Slk_D = Slk_3 = -0.488 + 0.167R - 0.026V - 0.02T$

$$Proj(Slk_D) = -0.488 - 3 \times \sqrt{(0.167)^2 + (-0.026)^2 + (-0.02)^2} = -1$$

Using RSS in Projection here

DPT Method



- Work with Deterministic Numbers
  - Assume deterministic corner $(V, T) = (0{,}0)$
  - So, deterministic = mean
- Start at Data End of the Test (D) with projected Slack: $Proj(Slk_D) = -1$
- Suppose CPPR cutoff = 0. So, start peeling paths for this test.
- Walk back to first "merge" point 3.
- Late Mode Incoming Deterministic Propagated ATs:

$AT_1 + d_1 = 100$ ← Diff = 11 = 111-100

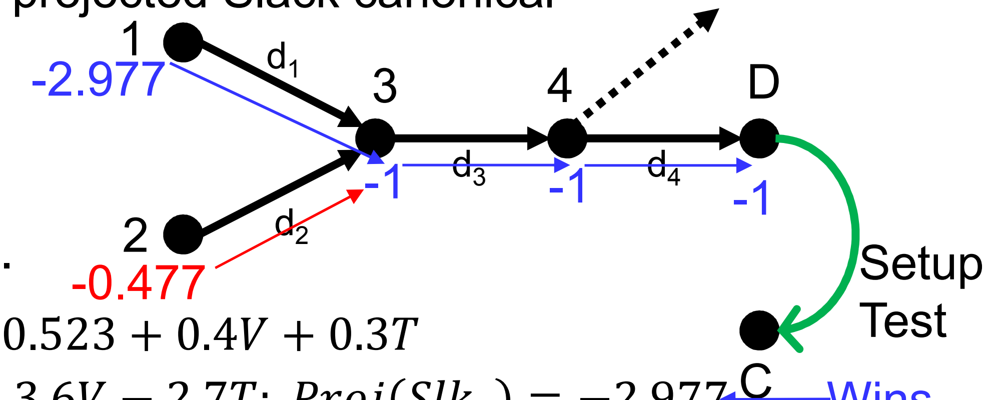
$AT_2 + d_2 = 111$ ← 2 Wins

- 2 wins because it has higher propagated AT.
  - Copy slack (-1) from 3 and back-propagate critical path to 2.
- Compute "slack" at 1 = sink_slack + AT_diff = -1 + 11 = +10
  - Would not schedule 1 for later peeling since it exceeds CPPR cutoff = 0.
- Problems with DPT:
  1. Chooses 2 to be "critical" and assumes $Proj(Slk_2) = -1$. False slack continuity.
     - Will see this to be incorrect.
  2. Does not schedule 1 for future peeling because it "thinks" $Proj(Slk_1) = +10$.
     - Will see this to be incorrect also.

Proposed SPT Method

- Start at Data End of the Test (D) with projected Slack canonical

  $Proj(Slk_D) = -1$



- Suppose CPPR cutoff = 0.
  - So, start peeling paths for this test.
- Walk back to first "merge" point 3.
- Compute Edge-Slacks: $RAT_3 = 110.523 + 0.4V + 0.3T$

  $Slk_1 = RAT_3 - (AT_1 + d_1) = 10.523 - 3.6V - 2.7T;\ Proj(Slk_1) = -2.977$ ← Wins

  $Slk_2 = RAT_3 - (AT_2 + d_2) = -0.477 + 0V + 0T;\ Proj(Slk_2) = -0.477$ ← Peel later

- 1 wins because it has lower projected edge-slack = -2.977.
  - back-propagate critical path to 1.
- Schedule 2 for later peeling with projected edge-slack = -0.477 < 0.
- Note that DPT got both slacks at 1 and 2 wrong.
  - It picked the wrong winner (2). Even tightness probs on Stat MAX of ATs on node3 picks wrong winner (2).
- Also, no continuity in projected slack due to Stat MAX on ATs at node 3.
- <u>SPT advantages</u>:
  1. Chooses <u>slack</u>-critical inputs first at a merge point. Finds critical paths sooner.
  2. Computes <u>correct</u> path-slack canonical as it walks backwards.
     - Slack at the starting node of a peeled path is precisely the projection of the path-based slack canonical computed using path-based AT along that path.
  3. Makes CPPR check against "cutoff" more meaningful.

Results

- **CPPR** (Common Path Pessimism Removal) implemented in a GBA (Graph or Block-Based Analysis) Statistical Timer
  - cutoff: Only analyze tests with original slack (OS) < specified value. Also, during path-tracing, schedule sub-critical inputs at a merge point with slack < specified value.
  - PPT (Paths Per Test): Stop further path peeling if number of paths currently peeled exceeds specified number.
  - Implemented both DPT and SPT as Path-tracing methods within CPPR.
- **Exhaustive CPPR**: unlimited cutoff and PPT.
  - Analyze all tests and peel all paths.
- **WS** (Worst Post-CPPR Slack) for a test.
- **WPI** (Worst Path Index): The path number at which WS was found for that test.
- **CPPR Safety**: PPT $\geq$ max WPI over all tests
  - smaller WPI is better since CPPR can be run with smaller PPT; hence faster.
- DPT vs SPT compare on 4 industrial designs (Exhaustive CPPR)

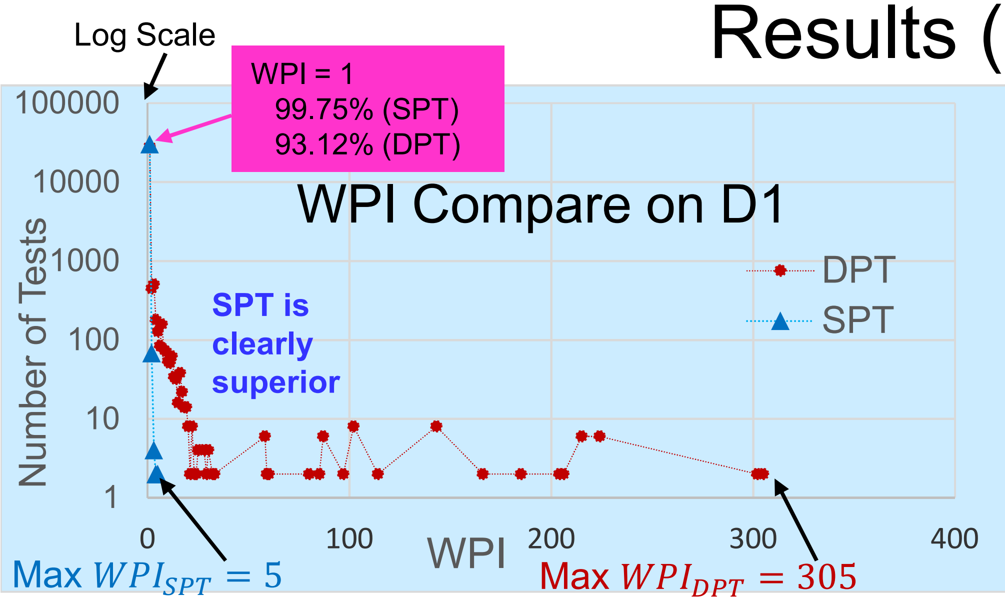
Design (# gates)	Number of Tests			
	Total	$WPI_{SPT} > WPI_{DPT}$	$WPI_{SPT} < WPI_{DPT}$	$WPI_{SPT} = WPI_{DPT}$
D1 (13K)	30K	12	2K	28K
D2 (0.5K)	2K	0	0.1K	1.9K
D3 (353K)	806K	451	64K	741K
D4 (12K)	25K	22	2K	23K

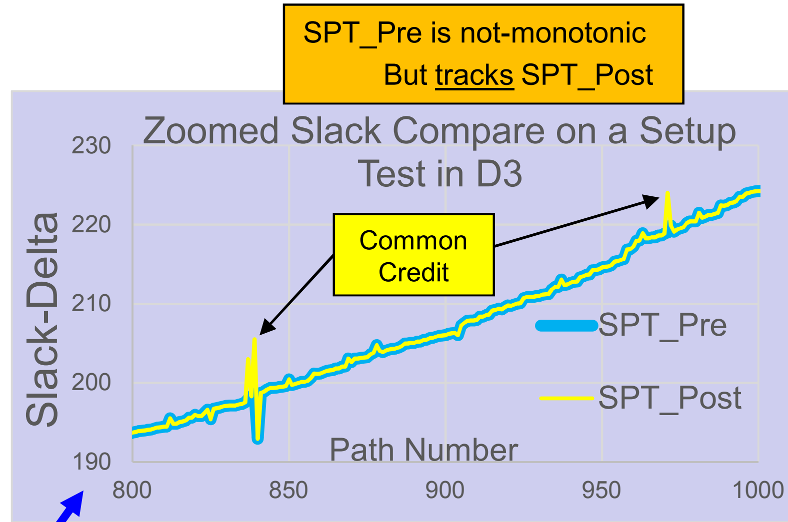
Max WPI-diff = 3

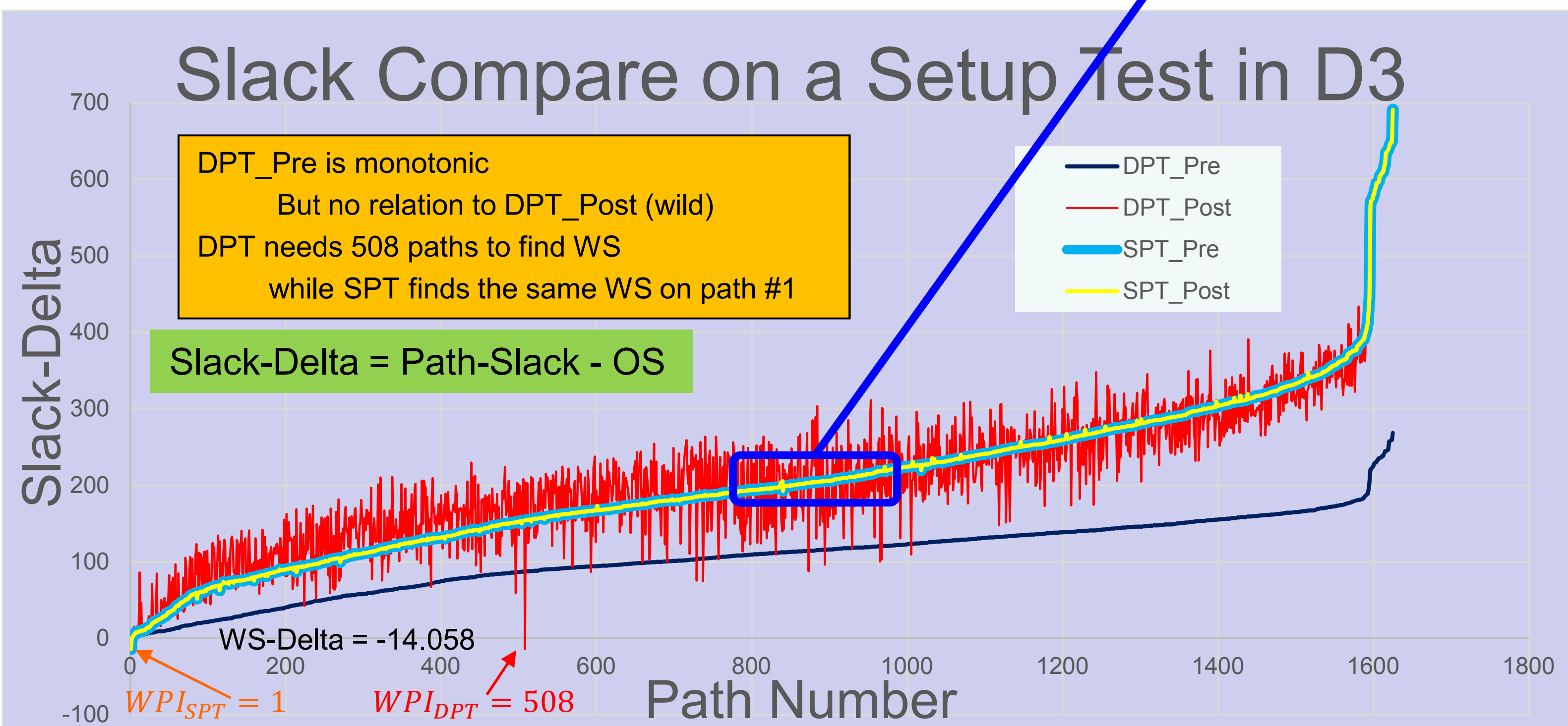
SPT is clearly superior

Design	Max WPI_{DPT}	Max WPI_{SPT}
D1	305	5
D2	4	1
D3	8364	31
D4	114	9

Results (Contd.)







Final Thoughts

1. SPT finds post-cppr critical paths sooner than DPT
   - SPT has smaller WPI
   - Can run CPPR with much smaller PPT; hence faster.
2. Running CPPR with cutoff = 10 on design D3
   - 99,889 tests were analyzed
   - 124 (i.e. 0.124%) of these showed $WS_{DPT} > WS_{SPT}$; so CPPR with DPT is unsafe when run with a cutoff.
     a. 9 of these were Setup Tests with diff $WS_{DPT} - WS_{SPT} \epsilon [13.038, 33.711]$ ps
     b. 115 of these were Hold Tests with diff $WS_{DPT} - WS_{SPT} \epsilon [0.0037, 13.083]$ ps
   - DPT in all except 15 out of 124 of these tests peeled <u>only</u> 1 path because all sub-critical branches weren't scheduled for future peeling since their "pathSlack" exceeded the given cutoff value.
3. In Exhaustive CPPR, SPT is about 2X slower than DPT
   - Not surprising considering the extra stat calculations needed in SPT vs DPT
4. In a "Production" CPPR run on D3, SPT has been seen to be 30% faster than DPT due to much smaller PPT budget.
   - Much fewer paths peeled by SPT due to much smaller WPI.