Efficient Parasitic Interconnect Insertion for Timing Analysis

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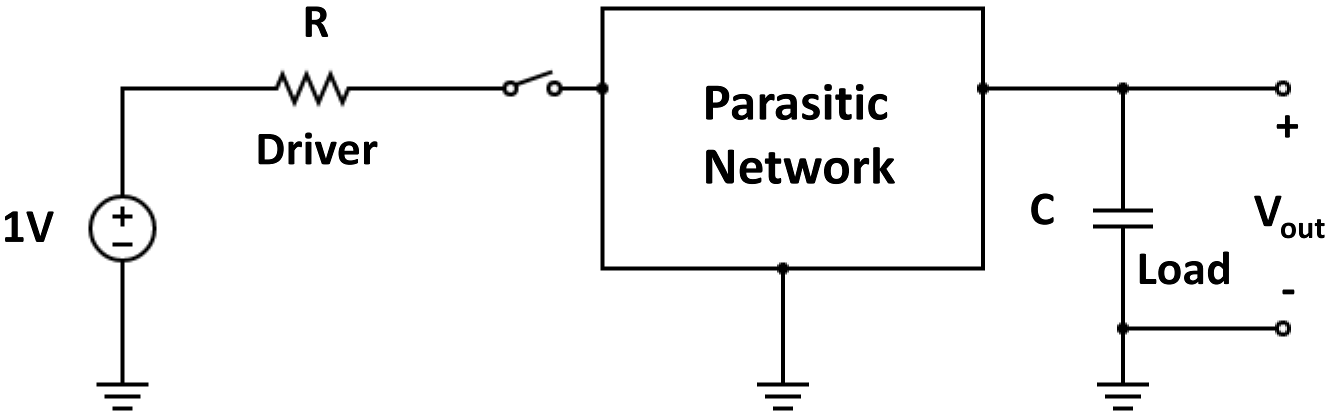
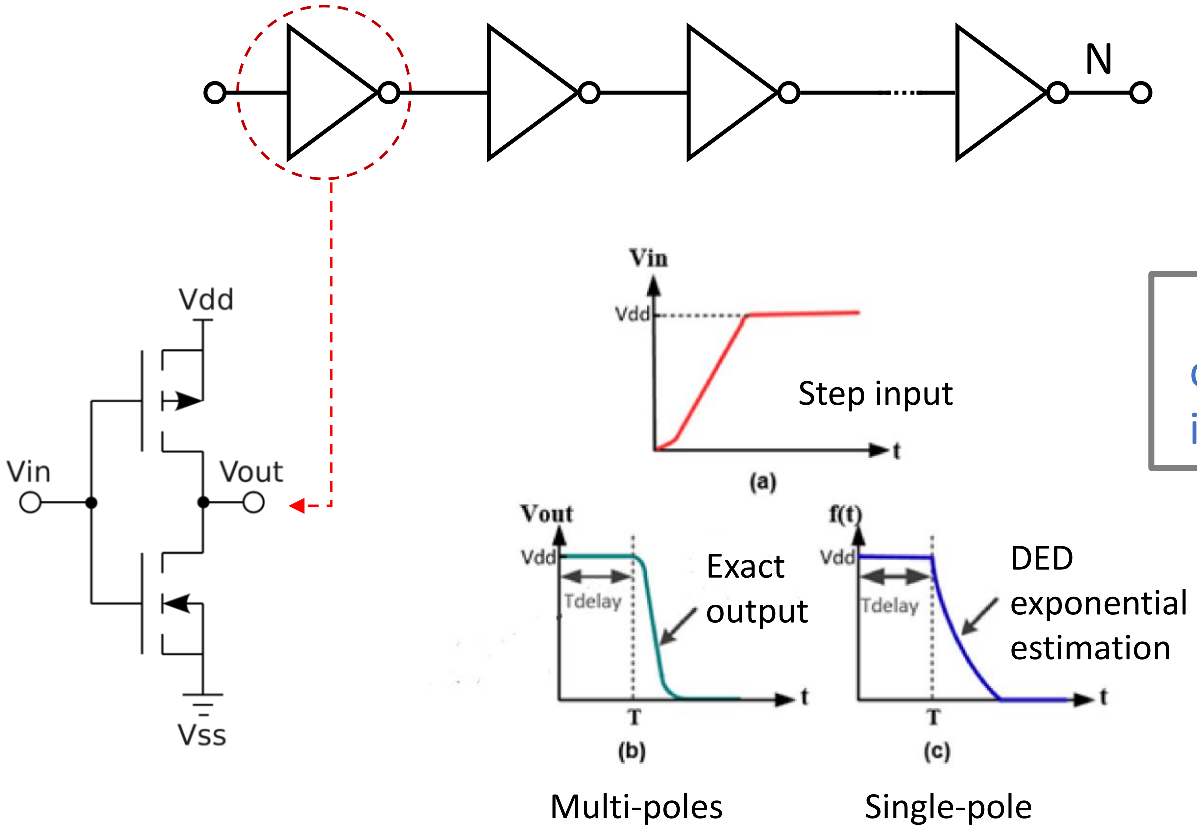
Parasitic Interconnect Network



Delayed Elmore Delay Macromodel (DEDM)



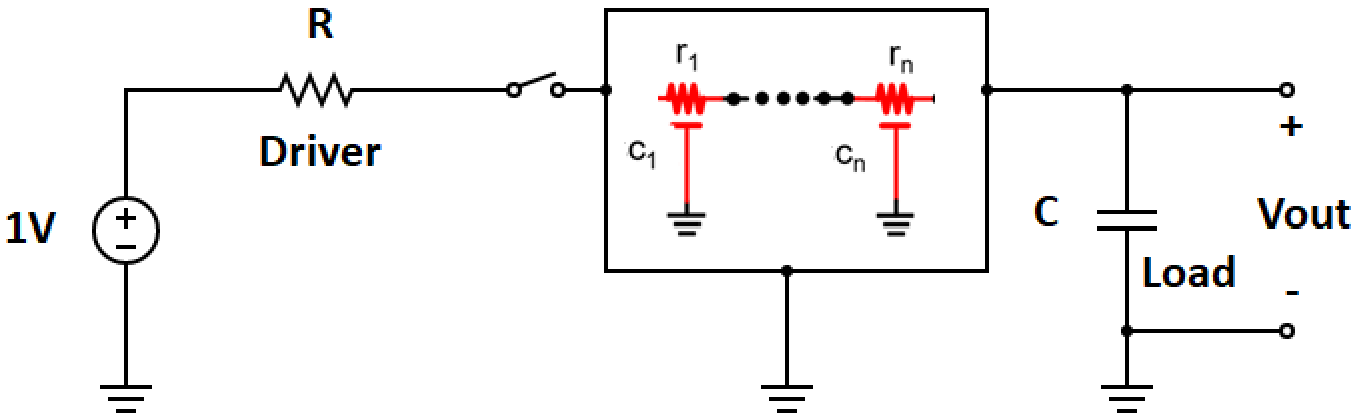
DEDM filters out small time constants and finds the post inertial delay dominant pole

DEDM Speed & Accuracy

Circuit		DEDM								
		Delay Accuracy	Signal Error @:			Speedup Vs:			Accuracy Vs:	
		90% output	10% output	τ	90% output	SPICE	1-pole model	ESCM*	1-pole model	ECSM
Inverter		99%	3%	4%	1%	1.2k	0.7	6x	6x	2x
NAND		98%	2%	3%	2%	1.0k	0.7	4x	6x	3x
Half adder (Carry)	LS	97%	2%	1%	2%	1.8k	0.6	9x	15x	3x
	MS	97%	1%	1%	3%	1.8k	0.6	9x	18x	4x
	HS	95%	3%	3%	3%	1.8k	0.6	9x	20x	4x
Flip flop		96%	4%	2%	2%	1.3k	0.6	-	25x	-

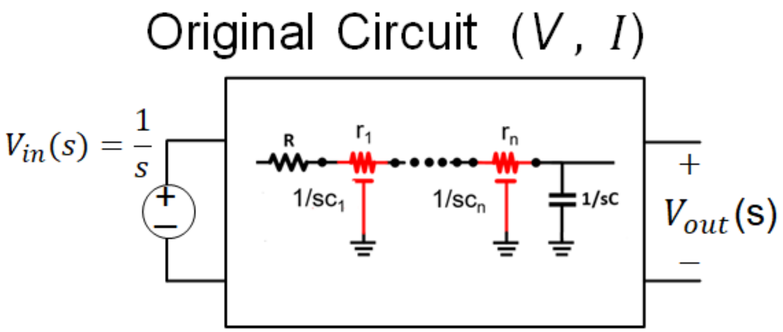
*ECSM (Effective Current Source Macromodel: Reference): S. Nazarian, H. Fatemi, and M. Pedram, “Accurate timing and noise analysis of combinational and sequential logic cells using current source modeling,” *IEEE Transactions on Very Large Scale Integrated Systems*, vol. 19, no. 1, pp. 92–103, 2011.

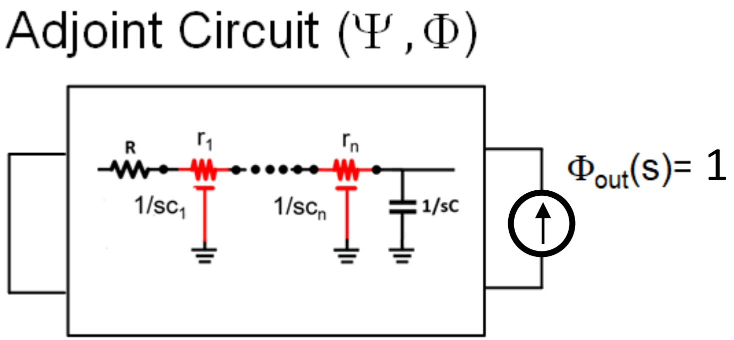
Parasitic Interconnect Ladder



Several small r's and c's form an interconnect ladder

Adjoint Circuit

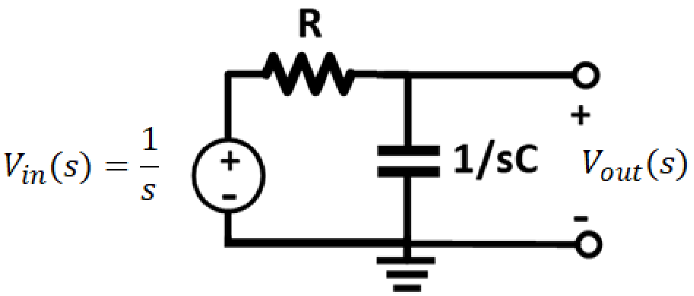




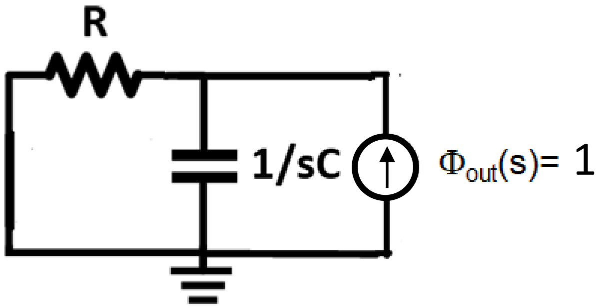
The adjoint circuit is topologically equivalent to the original circuit

Adjoint Network Theory*

Original circuit:



Adjoint circuit:



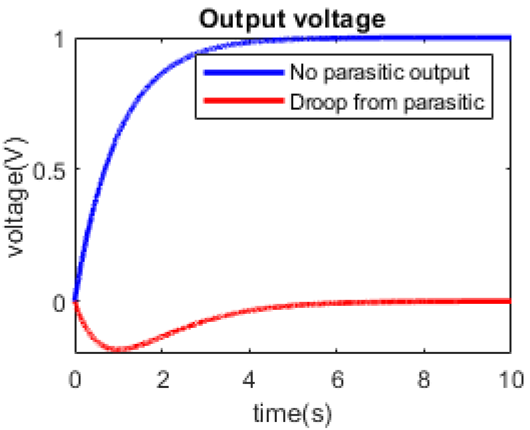
$$\delta V_{out}(s) = \sum_R i_R \phi_R \Delta R - s \sum_C v_C \psi_C \Delta C$$

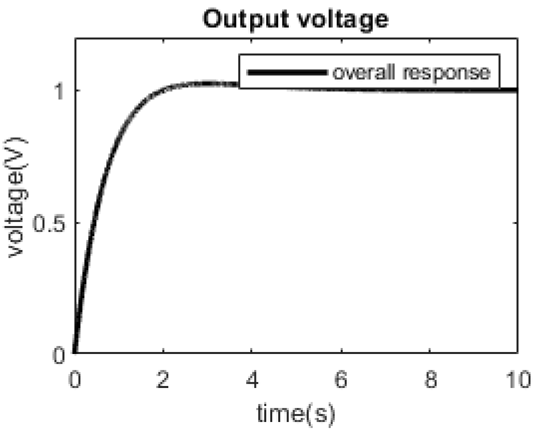
Adjoint & original circuit have the same poles (always!)

*S. W. Director and R. A. Rohrer, “The Generalized Adjoint Network and Network Sensitivities,” *IEEE Transactions on Circuit Theory*, vol. 16, no. 3, pp. 318–323, 1969.

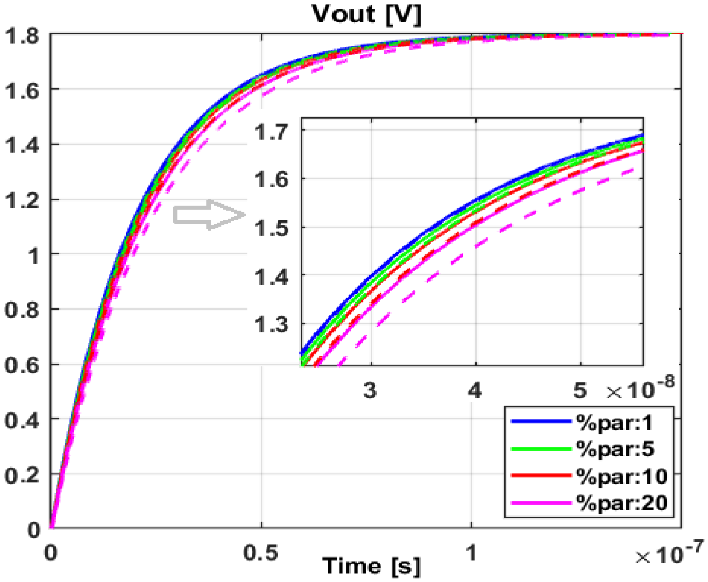
Adjoint Estimation of Timing Signal Droop

$$\delta v_{out}(t) = -\frac{t}{\tau} e^{-\frac{t}{\tau}} \left[\frac{\sum c}{C} + \frac{\sum r}{R} \right]$$

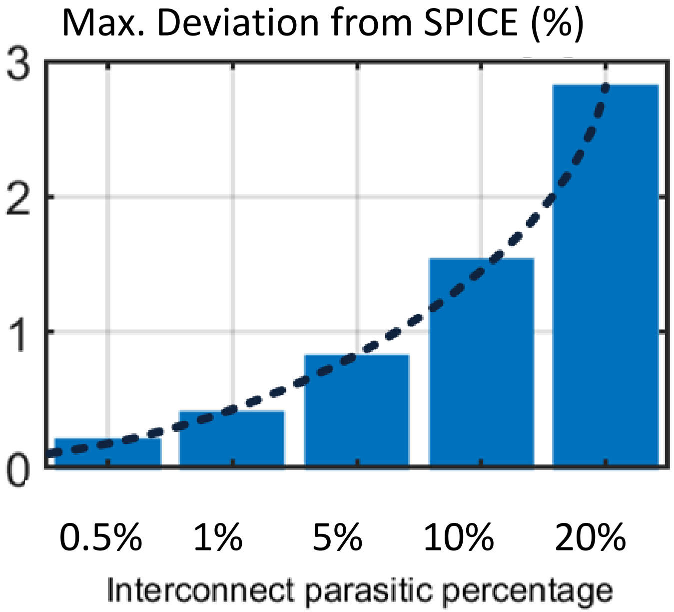




Adjoint Estimation Error



Solid lines: SPICE results
Dashed lines: Adjoint estimation

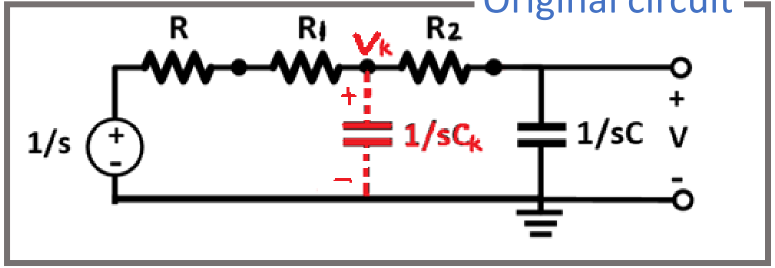


$$\left[\frac{\sum c}{C} + \frac{\sum r}{R}\right]$$

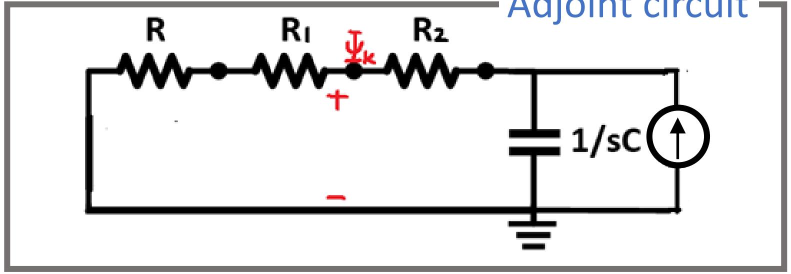
Refined Adjoint Estimation

- Include the parasitic r's in both original & adjoint circuits
- Only parasitic c's are treated as true parasitics
- Adjoint estimation of parasitic c's induced droop

Original circuit



Adjoint circuit



$$\delta V_{out}(s) = -s\sum_{k=1}^{n} v_k \psi_k c_k$$

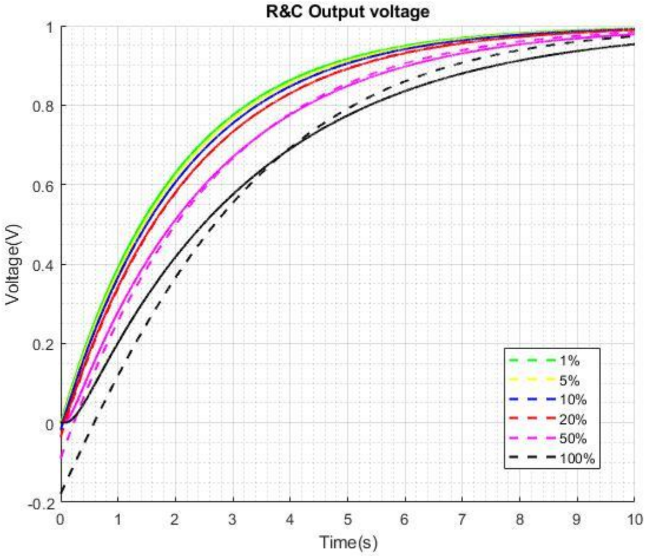
$$\delta V_{out}(t) = -[a_1 \frac{1}{T_1} e^{-\frac{t}{T_1}} + a_2 \frac{t}{{T_1}^2} e^{-\frac{t}{T_1}}]$$

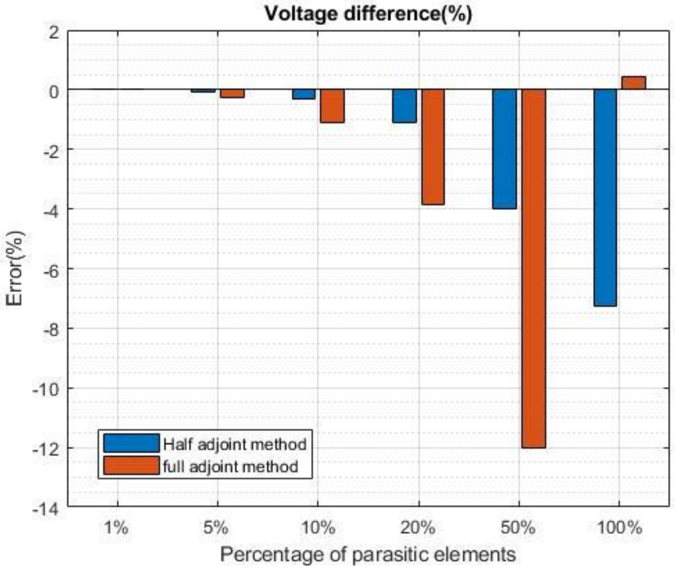
$$where\ T_1 = C\left(R + \sum r\right) \qquad a_1 = \sum_{k=1}^{n} [\frac{c_k(R + R_{1,k})R_{2,k}}{R + \sum r}]$$

$$a_2 = \sum_{k=1}^{n} [c_k(R + R_{1,k})] - a_1$$

$$R_{1,k} = \sum_{j=0}^{k-1} r_j \,,\, R_{2,k} = \sum_{i=k}^{n} r_i$$

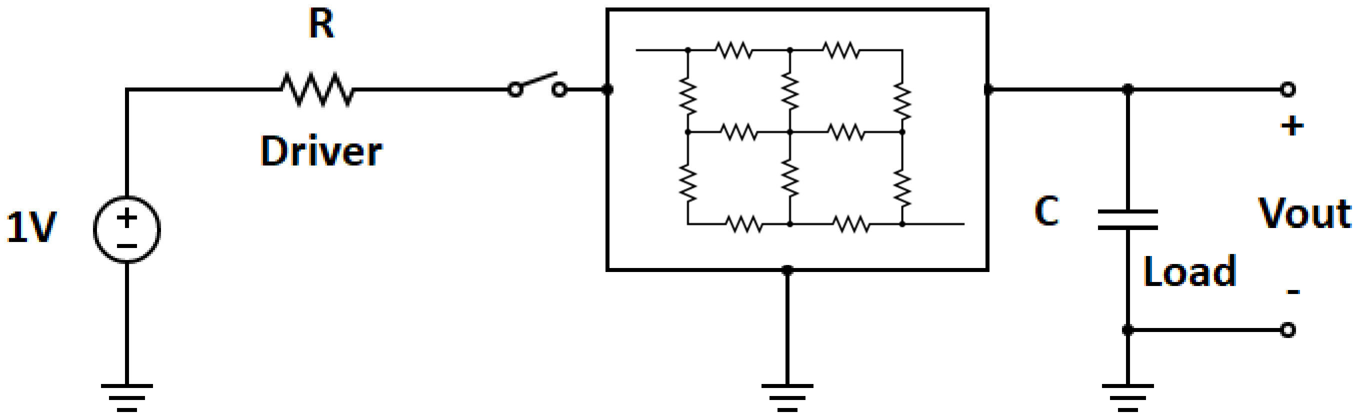
Refined Adjoint Error





Solid lines: SPICE results
Dashed lines: Adjoint estimation

Parasitic Meshes



Several parasitic r's form meshes, with parasitic c's connecting each node to ground (c's are not shown)

Alternative Approaches

- Asymptotic Waveform Estimation (AWE)
- Time Constant Equilibrium Reduction (TICER)
- Delayed Elmore Delay Macromodeling (DEDM)
- Adjoint Parasitic Insertion

Asymptotic Waveform Estimation (AWE)

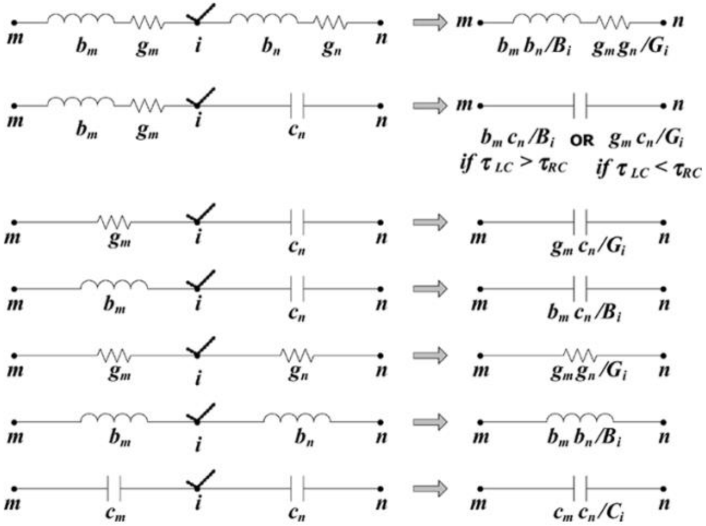
Pade Approximation

Moment expansion of the circuit transfer function: $H(s) = m_0 + m_1 s + m_2 s^2 + \cdots$

The first few moments determine the dominant poles and related zeros:

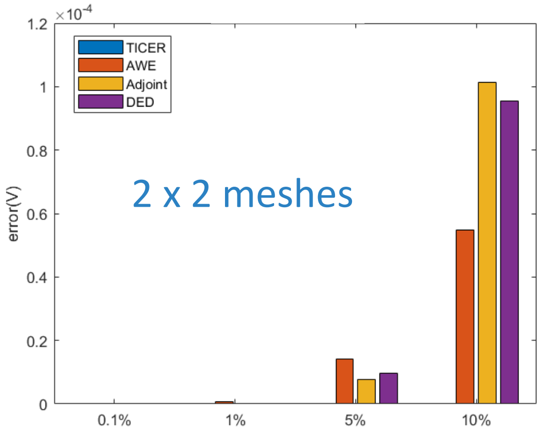
$$H(s) = m_0 + m_1 s + m_2 s^2 = \frac{\widehat{b_0} + \widehat{b_1} s}{1 + \widehat{a_1} s + \widehat{a_2} s^2}$$

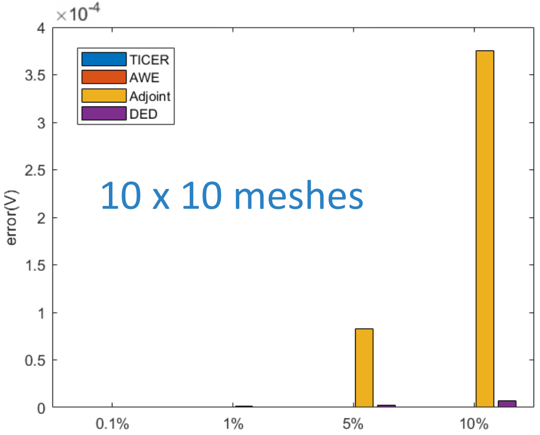
Time Constant Equilibrium Reduction (TICER)



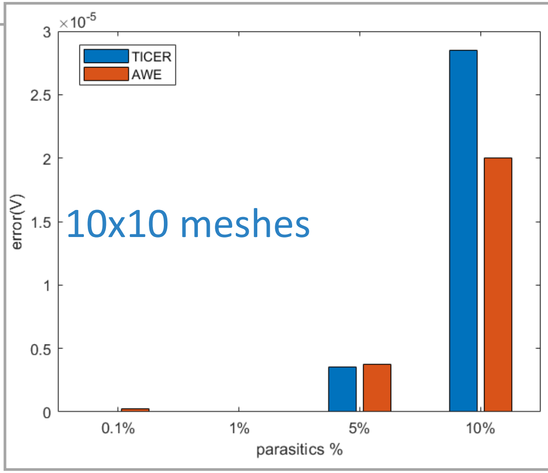
Eliminate nodes with small time constants, and distribute associated r's and c's to neighboring nodes

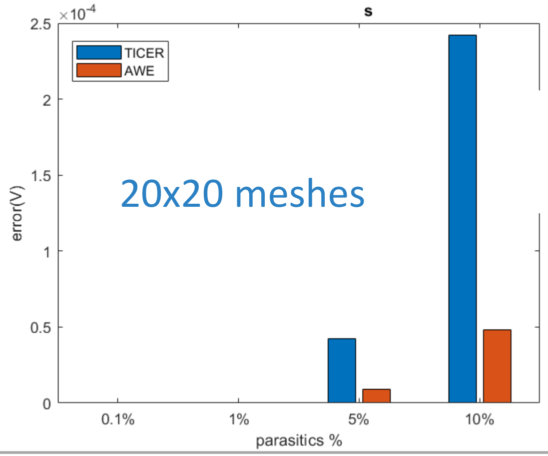
Simulation vs. Estimations: mean-squared error





Error vs. grid size





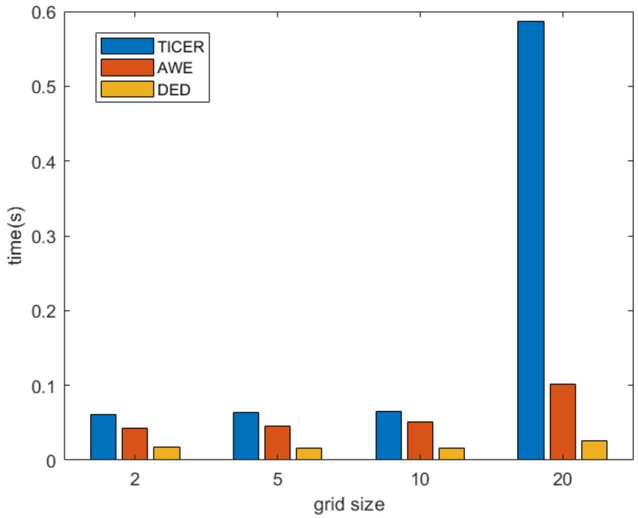
Error vs. parasitics

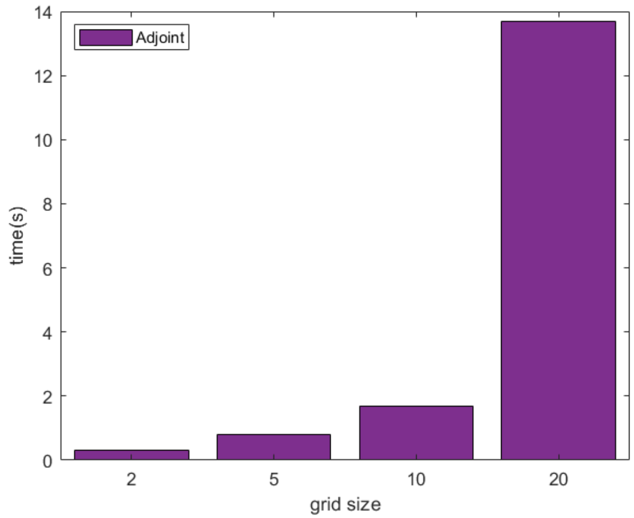
Adjoint is worse with increase in number of meshes

For very large circuits, AWE is superior

Simulation and Estimation

Runtime (MATLAB) vs. grid size





Adjoint runtime is unacceptably long, AWE & DED methods are faster.

Observations

- AWE
  - Is a superior approximation approach
    - Faster
    - More memory efficient
    - More accurate
- Insights from Adjoint
  - Start with coincident poles and root-locus
  - Original AWE presumed separate poles

Repeated-poles AWE: inspired by adjoint

Assume that the output expression has the following form for a step input:

$$f(t) = 1 - e^{-at} - Kte^{-at}$$

Then the frequency domain transfer function is

$$H(s) = 1 - \frac{s}{s+a} - K\frac{s}{(s+a)^2}$$

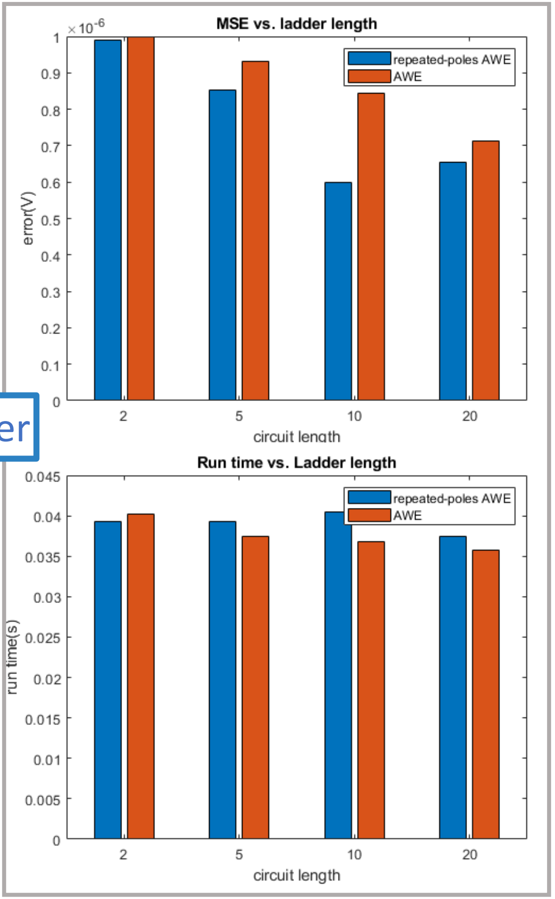
$$= \frac{(a-K)s + a^2}{(s+a)^2}$$

$$= m_0 + m_1 s + m_2 s^2$$

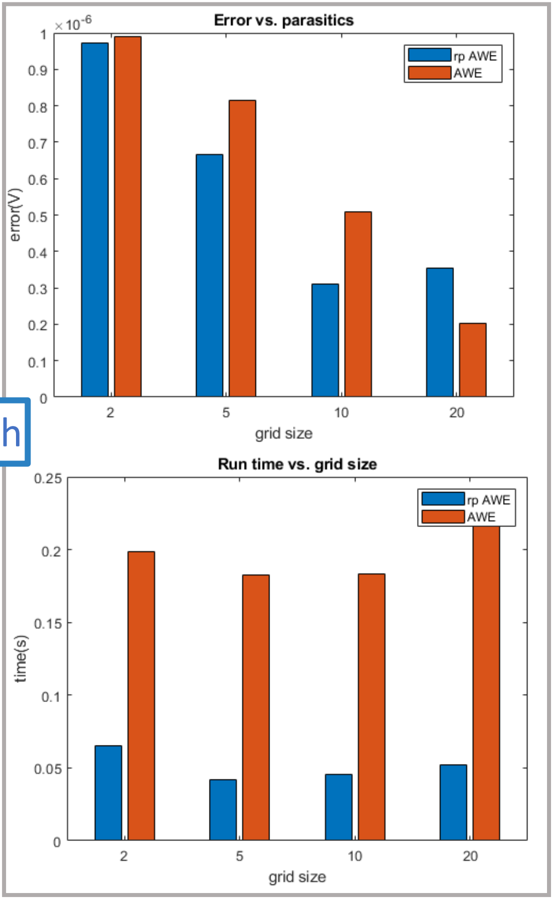
Solve for only two parameters: a and K

Simulation vs. Estimations

RC ladder



RC mesh



AWE + repeated poles has the same degree of error as AWE…

but entails less runtime.

Thank you!

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Research / Graduate Student — Yu Hong (SMU)

And a Request:

Please send realistic interconnect examples
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