Linearity of MAX Operation in Statistical Timing Analysis

Lizheng Zhang

Weijen Chen, YuHen Hu

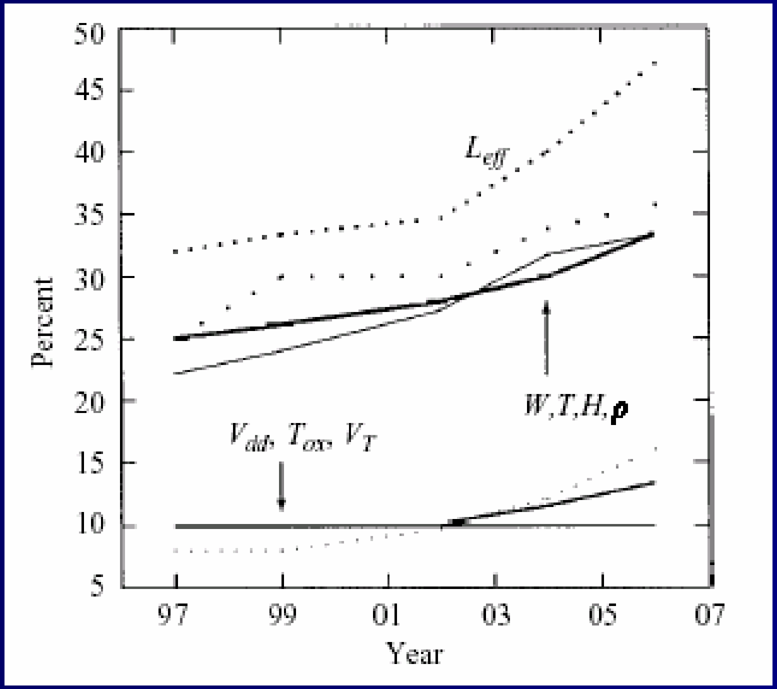
Charlie Chungping Chen

University of Wisconsin, Madison

Variation Sources

- Environmental Variations
  - Power Supply Uncertainty, Temperature fluctuation
- Processing Variations
  - Channel length, Doping density, Threshold voltage, Oxide thickness, Wire width and thickness, ……

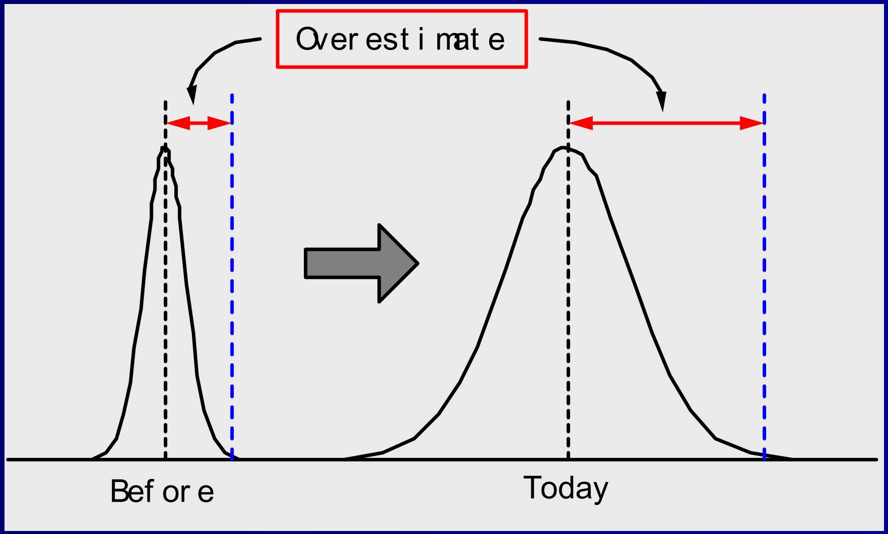
Variation Magnitude



- The percentage of the variation versus absolute value for parameters
  - 20~30% in today's technology
  - Larger in the future

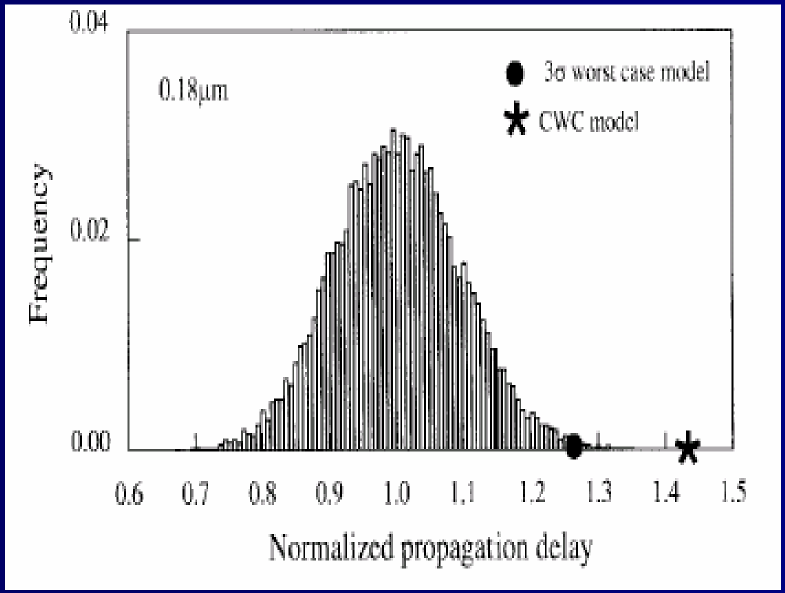
"Models of process variations in device and interconnect"
Duane Boning, MIT & Sani Nassif, IBM ARL.

Overestimation of Worst Case Analysis



- Pessimistic estimation:
  - It is very rare to have all gates behave as the worst case
  - Larger variation make overestimation more significant

Worst-case Timing Analysis Example



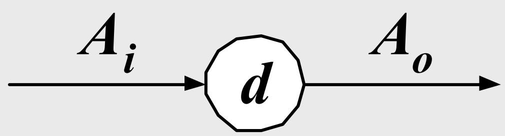
- 16-bit Adder
  - Longest path Delay
- Monte Carlo
  - $3\sigma = 1.25$
- Worst-case Timing
  - Delay=1.45
- Too **pessimistic**!
  - 20% overestimation

"Impact of Unrealistic Worst Case Modeling on the Performance of VLSI circuits in Deep Submicron Region CMOS Technologies"
A.Nardi, A.Neviani, E.Zanoni,M.Quarantelli, *IEEE '99*

Block-Based SSTA

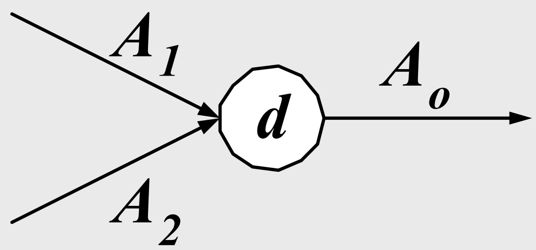
- **Signal arrival time** propagates from gate to gate in the circuit timing graph without looking back into its history
- Elemental Operations:
  - ADD and MAX
- Linear to size of circuit

ADD/MAX Operation



$$A_o = A_i + d$$

ADD Operation

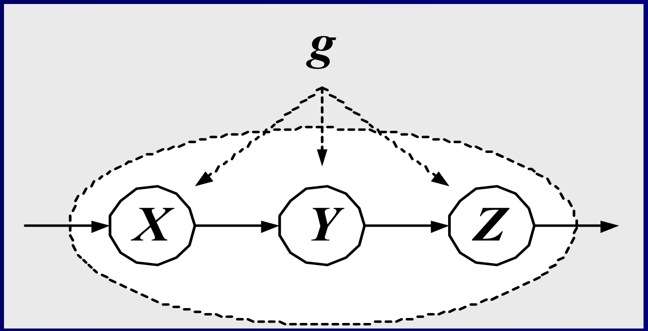


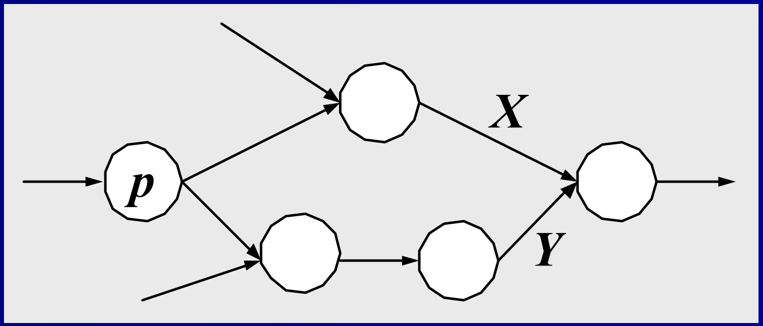
$$A_o = \max(A_1, A_2) + d$$

MAX Operation

"Statistical timing analysis using bounds and selective enumeration" *Agarwal, A.; Zolotov, V.; Blaauw, D.T.;*
TCAD, Vol. 22(9) , Sept. 2003, P1243 - P1260

Difficulty(1)--Correlation





- Global Correlation
  - *X,Y,Z* are all dependent on *g*
- Path Correlation
  - *X* and *Y* both dependent on *p*

Difficulty (2) – MAX

- It is **non-linear**
  - MAX of Gaussian random variables will **not** be Gaussian
- The difficulty comes from the fact that we want to **preserve correlation** information after non-linear MAX

Linear MAX Approximation

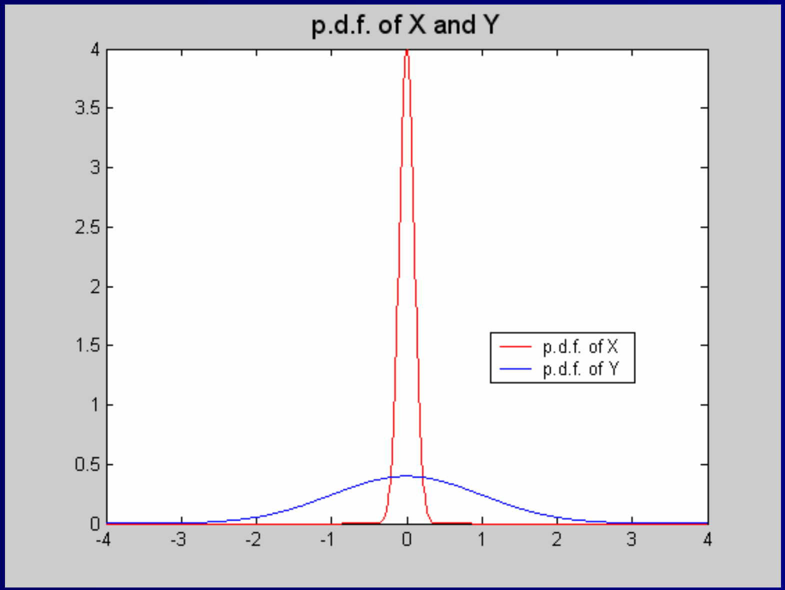
- MAX operation is approximated by a linear mixing operator:

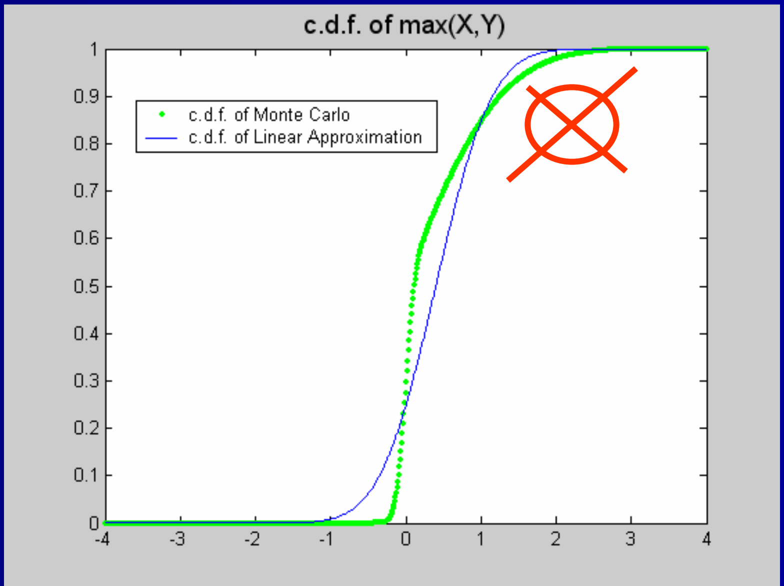
$$max(X, Y) \approx QX + (1 - Q)Y + \Delta$$

  - Matching mean, variance and covariance

"First-order incremental block-based statistical timing analysis" *Visweswariah, C.; Ravindran, K.; Kalafala, K.; Walker, S.G.; Narayan, S.;* DAC'04. June 7-11, 2004, 331 - 336

It is risky!!



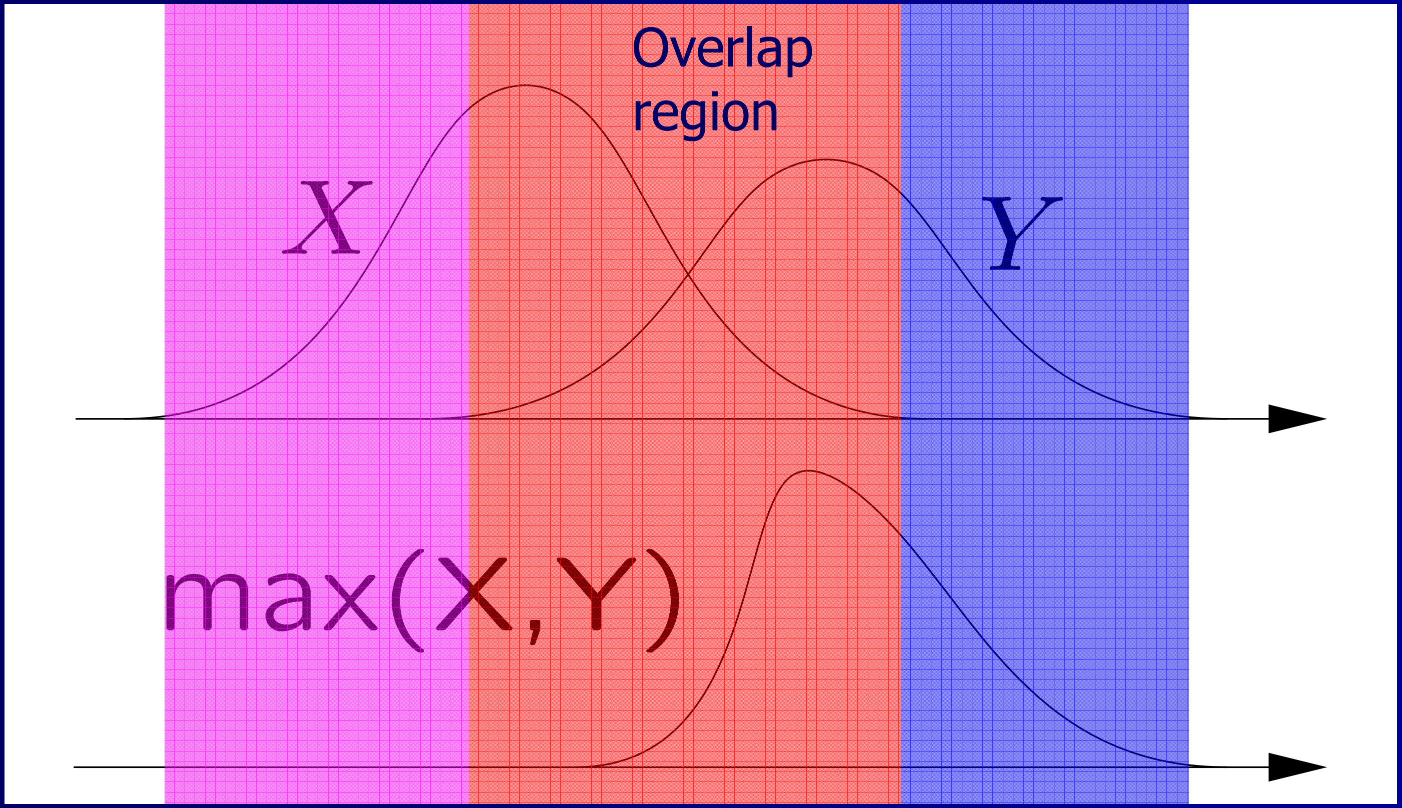


- **Risky**: Underestimate delay in high confidence level

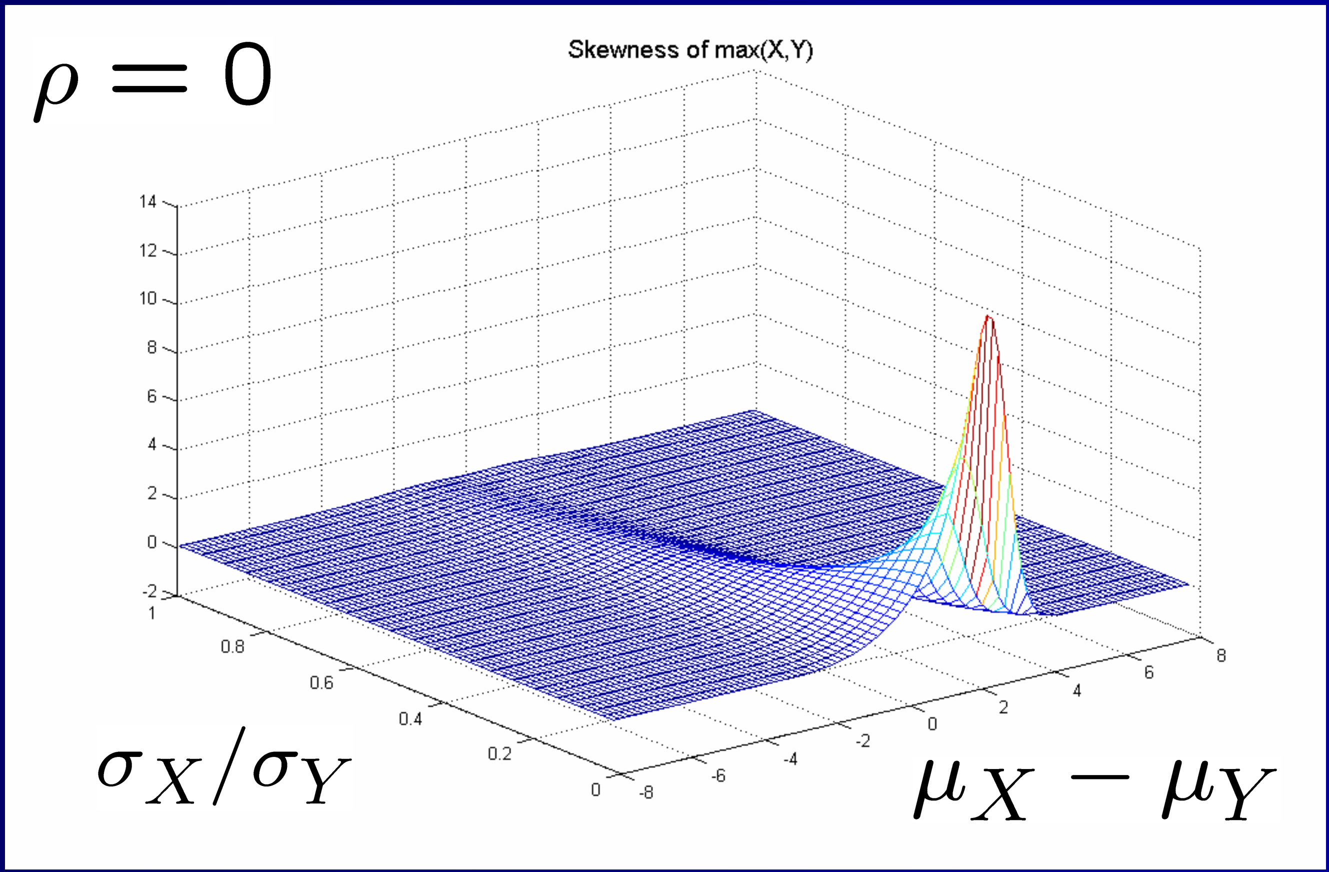
Non-Linearity of MAX

- Linearity of the MAX depends on the inputs
- When inputs are Gaussian, non-linearity of MAX operator is equivalent to the Gaussianity of the output
- Is skewness a good choice to decide the Gaussianity?
  - Pro: It can be computed analytically
  - Con: Generally zero-skew does NOT mean Gaussian.

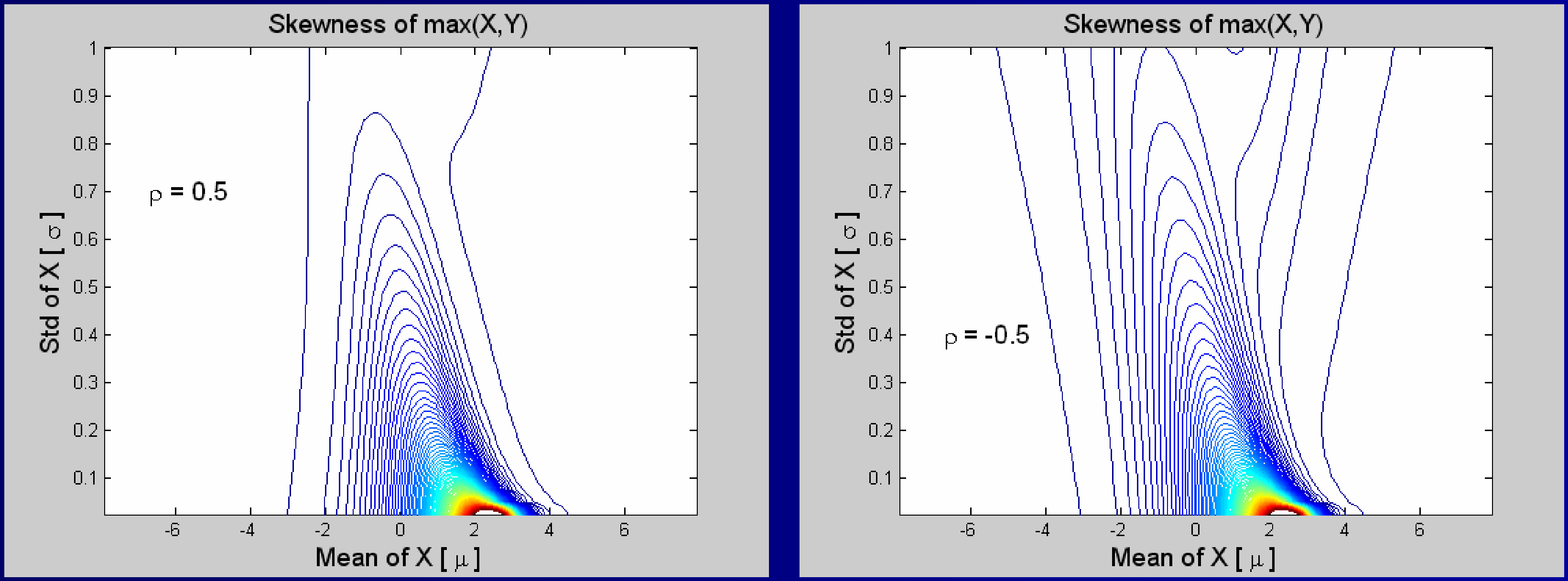
Skewness of MAX output



Positive Skewness



Skewness and Correlation



- More positive correlation means small skewness and less non-linearity

Proposed Solution for non-linear MAX

- Determine when the non-linearity of MAX using skewness
- Using Linear approximation when MAX is linear
- Using Bounding approach when MAX is non-linear

Bounding Theorems

- For non-decreasing functions
- Function upper-bounds ⇔ CDF upper bounds
- With the same confidence level

Theorem 1 *For any random variable X and monotonic non-decreasing functions* $Z = \varphi(X)$ *and* $\hat{Z} = \psi(X)$ *which are defined at* $-\infty < X < +\infty$*, if* $Z \leq \hat{Z}$ *for all* $a \leq X \leq b$*, then the* $c.d.f.$ *of* $\hat{Z}$ *will be an upper bound of the* $c.d.f.$ *of Z within interval of* $[\varphi(a), \psi(b)]$*, i.e.*

$$F_Z(z) \geq F_{\hat{Z}}(z) \quad \textit{if} \quad \varphi(a) \leq z \leq \psi(b)$$

Theorem 2 *For any random variable X and monotonic non-decreasing functions* $Z = \varphi(X)$ *and* $\hat{Z} = \psi(X)$ *which are defined at* $-\infty < X < +\infty$*, if* $Z \leq \hat{Z}$ *for all* $a \leq X \leq b$*, then the probability to have* $\varphi(a) \leq Z \leq \psi(b)$ *will be no less than the probability to have* $a \leq X \leq b$*:*

$$P\{\varphi(a) \leq Z \leq \psi(b)\} \geq P\{a \leq X \leq b\} \qquad (1)$$

Apply to MAX Operation

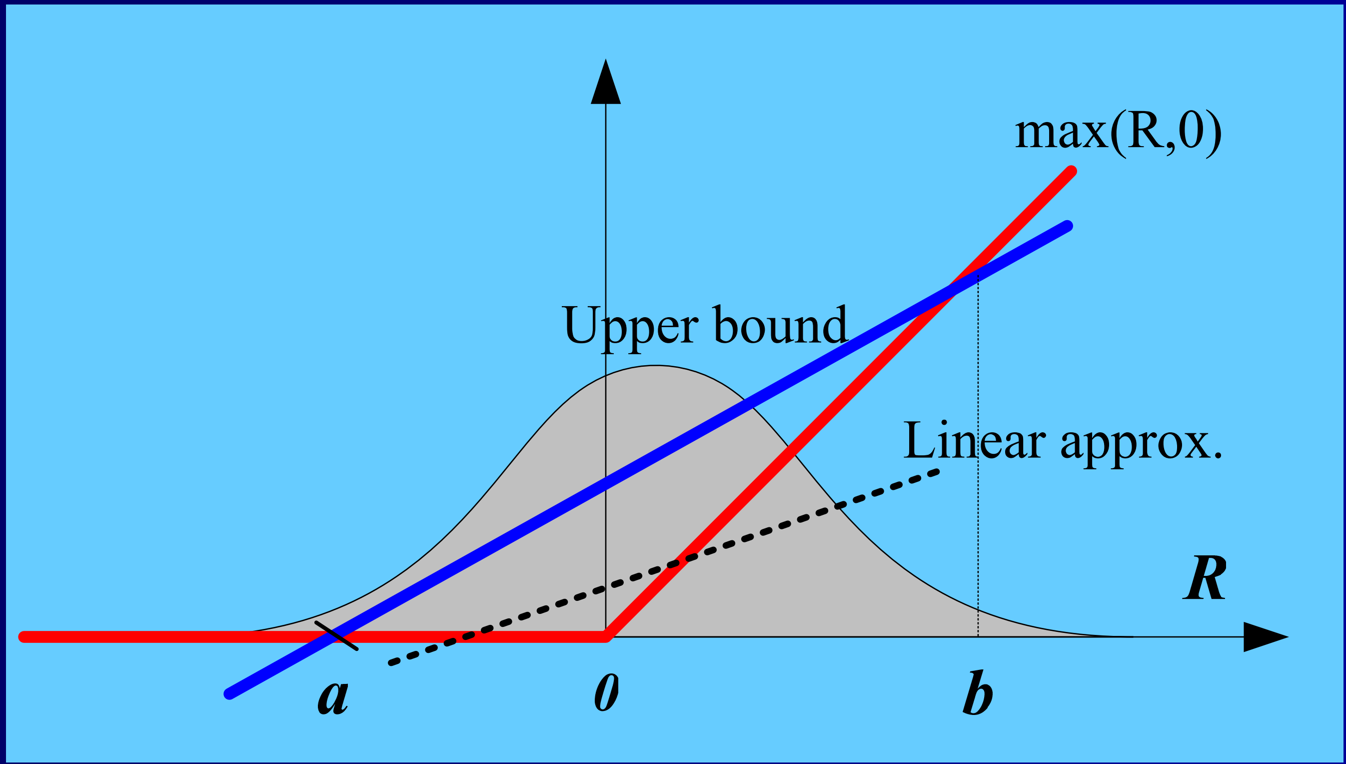
For any random variables X and Y and $R = X - Y$, the following transformation will always be true:

$$max(X, Y) = max(X - Y, 0) + Y = max(R, 0) + Y$$

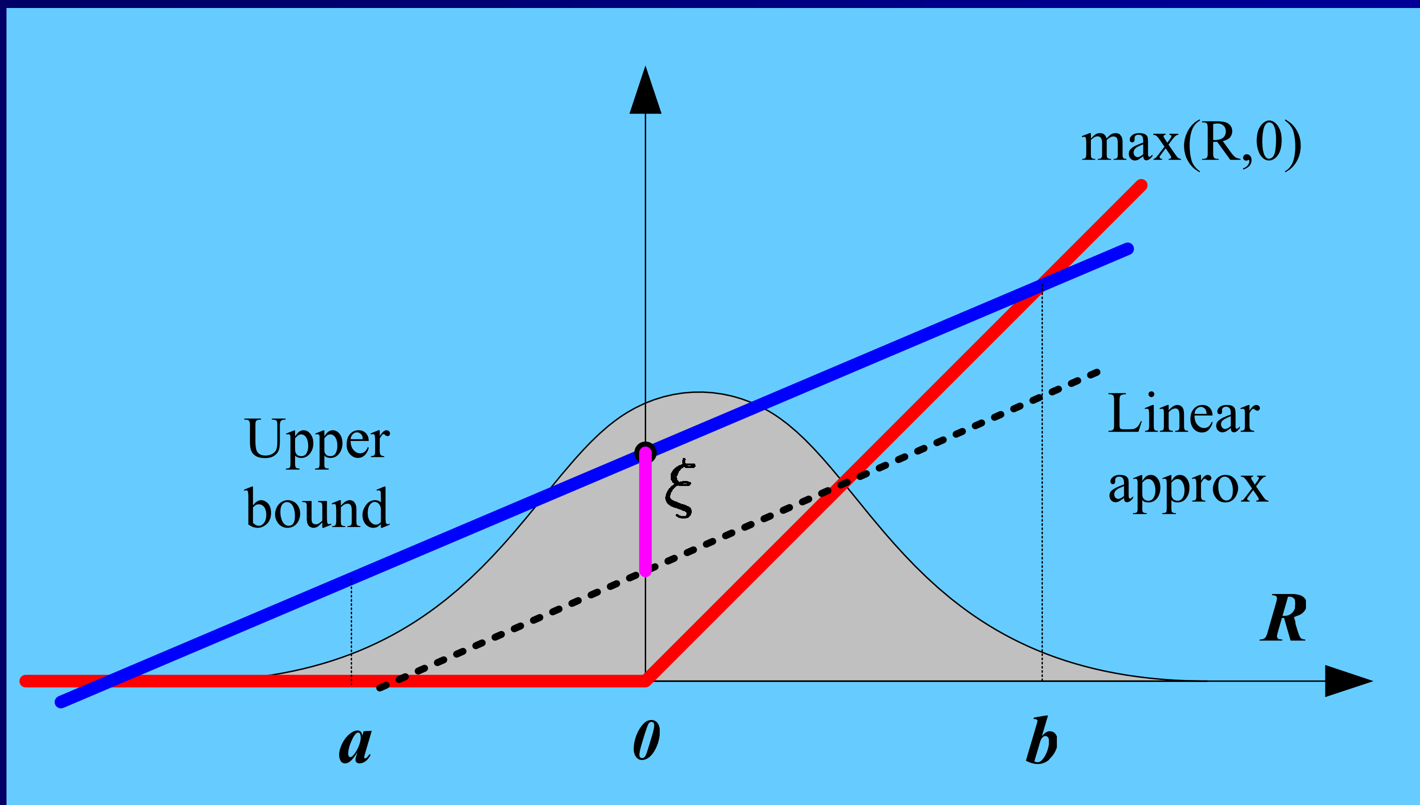
So it is equivalent to approximate $W = max(R, 0)$ linearly with $\widehat{W} = Q \cdot R + \Delta$ if the linear approximation for MAX is considered:

$$max(X, Y) \approx QX + (1 - Q)Y + \Delta$$

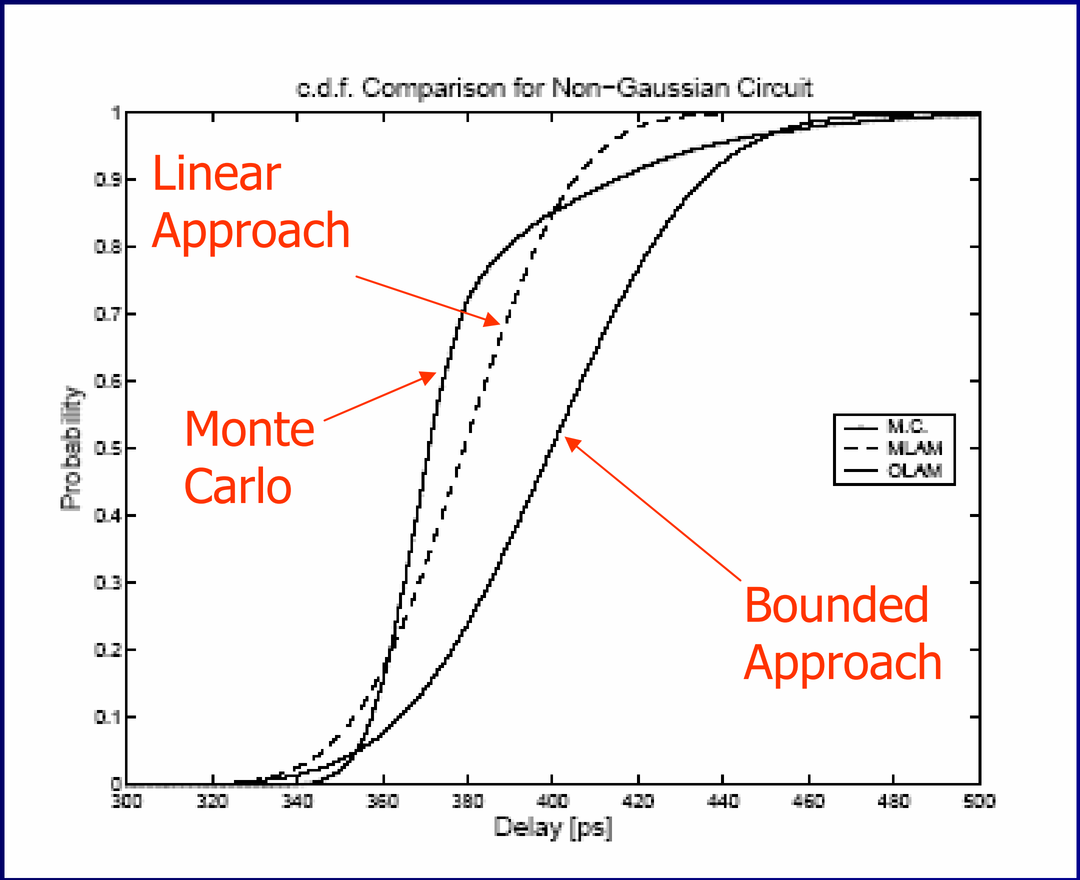
Graphical Illustration



Conditional Mean Modification



Non-Gaussian Circuits



Gaussian Circuits

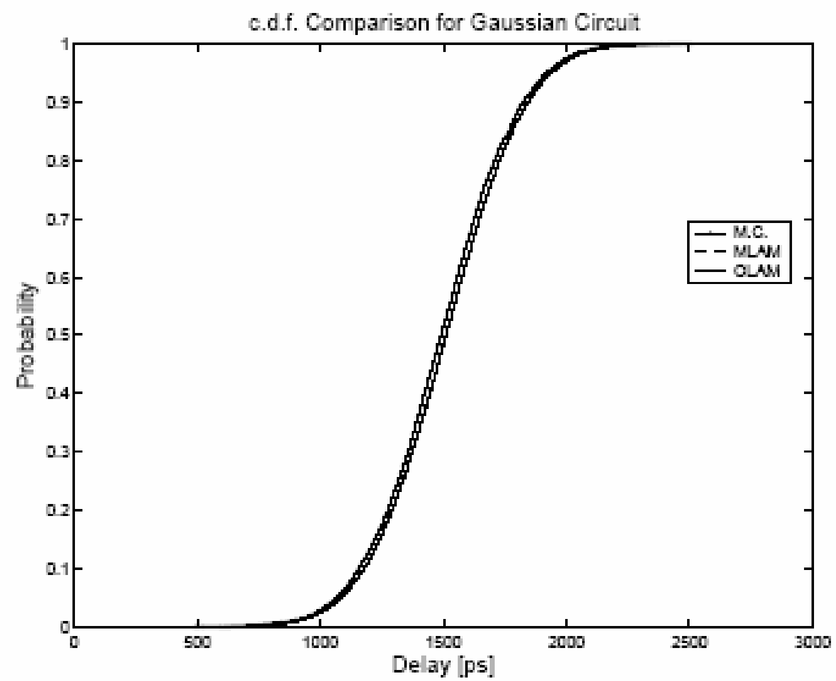


Fig. 7: Comparison of $c.d.f.$ for Gaussian Circuit

Conclusions

- Block-based statistical timing faces the difficulty to preserve correlation information through non-linear MAX operation
- It is risky to apply linear approximation for MAX unconditionally
- Skewness is a good parameter to measure the linearity of the MAX operator
- Based on the skewness, a new algorithm for block-based timing analysis is developed to safely take advantage of linear MAX approximation while avoiding the risk it might has