Sensitivity-Based Gate Delay Propagation in Static Timing Analysis

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Outline

• Conventional techniques
• Energy-based gate delay propagation (E4)
• Sensitivity-based gate delay propagation (SDP)
• SDP application in crosstalk target identification
• A hybrid algorithm (SDP+E4)
Overview

• In many STA tools, large errors can be observed when crosstalk noise is present.

• STA should not be optimistic; it is desirable that it is not too pessimistic either.
Introduction

- Gate delay propagation
- Interconnect delay propagation
Conventional Waveform Modeling

- Equivalent linear waveform, $\Gamma_{\text{eff}}^{i_{\text{in}}}$:
  - $\Gamma_{\text{eff}}^{i_{\text{in}}}$ - Arrival time: The latest $0.5V_{dd}$ crossing point of the noisy input waveform.
  - $\Gamma_{\text{eff}}^{i_{\text{in}}}$ - Transition time (calculated based on an effective slope for the input waveform)
Problem Definition

- The goal is to find an equivalent input line, $\Gamma_{in}^{eff}$, such that when it is applied to the input of a gate, it generates an output waveform that matches the actual waveform in terms of its arrival time and transition time.
Existing Approximation Techniques

• Point-based approximations
  - P1: $I_{\text{eff}}$ transition time is set to the time from $0.1V_{\text{dd}}$ to $0.9V_{\text{dd}}$ crossing points of the noiseless input waveform
  - P2: $I_{\text{eff}}$ transition time is set to the time from $0.1V_{\text{dd}}$ to $0.9V_{\text{dd}}$ crossing points of the noisy input waveform
  - P1 and P2 set the $0.5V_{\text{dd}}$ of $I_{\text{eff}}$ to the latest $0.5V_{\text{dd}}$ crossing point of the noisy input waveform

• Least Square approximations
  - LSF3: $I_{\text{eff}}$ is the pure least square fit of the noisy input waveform
  - WLS5: Performs weighted least square fitting to get $I_{\text{eff}}$

• Energy-based (E4) approximation
WLS5 Calculation Steps (Step I)

- Find the derivative for the noiseless input:
  \[ \rho_{\text{noiseless}}(t) = \frac{\partial v_{\text{out}}^{\text{noiseless}}(t)}{\partial v_{\text{in}}^{\text{noiseless}}(t)} \]

- Calculate the noiseless critical region \([t_{\text{noiseless first}}, t_{\text{noiseless last}}]\)

- \(\rho_{\text{noiseless}}\) is non-zero only for points in the noiseless critical region; otherwise it is set to zero
WLS5 Calculation Steps (Step II)

- Determining $\Gamma_{\text{eff \, \text{in}}}^\text{as}$:

$$ \text{Min} \sum_{k=0}^{P-1} \{ \rho^{\text{noiseless}}(t_k)(v_{\text{in}}^{\text{noisy}}(t_k) - (a \times t_k + b))^2 \} $$

![Graph showing time points and measurements](image)
Energy-Based Technique (E4)

- Pass $I_{\text{in}}^{\text{eff}}$ through the latest $0.5V_{\text{dd}}$ crossing point of the noisy voltage waveform.
- Slope is then selected such that the area, which is encapsulated by that line and straight lines $v_1(t) = 0.5V_{\text{dd}}$ and $v_2(t) = V_{\text{dd}}$, is equal to the area surrounded by the noisy input and lines $v_1$ and $v_2$. 

![Graph showing energy-based technique](image)
E4 Pessimism

• If the noisy input waveform has multiple $0.5V_{dd}$ crossing points, then E4 can become overly pessimistic.
Sensitivity-Based Gate Delay Propagation

• Considers the noisy critical region, $[t_{\text{noisy first}}, t_{\text{noisy last}}]$
SDP Calculation Steps (Step I & II)

- **Step I:** Find $\rho^{\text{noiseless}}$ (similar to Step I of WLS5).
- **Step II:** Estimate $\rho^{\text{eff}}$, the derivative of the output to the noisy input.
- For every $t_i \in [t_{\text{noisy first}}, t_{\text{noisy last}}]$, find $t_j \in [t_{\text{noisy first}}, t_{\text{noisy last}}]$ such that $v_{\text{in noisy}}(t_i) = v_{\text{in noiseless}}(t_j)$
- Next set $\rho^{\text{eff}}(t_j) = \rho^{\text{noiseless}}(t_j)$

![Graph showing noisy and noiseless input over time](image)
SDP Calculation Step III

- Find $\Gamma_{in}^{eff}$ with coefficients $a$ and $b$:

$$\text{Min} \sum_{k=0}^{P-1} \left\{ \rho^{eff}(t_k)(v_{in}^{noisy}(t_k) - (a \times t_k + b))^2 \right\}$$
## Accuracy Comparison

<table>
<thead>
<tr>
<th>Method</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max</td>
<td>Avg</td>
<td>Max</td>
</tr>
<tr>
<td>P1</td>
<td>81.3</td>
<td>29.3</td>
<td>134.2</td>
</tr>
<tr>
<td>P2</td>
<td>82.7</td>
<td>24.5</td>
<td>144.5</td>
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<tr>
<td>LSF3</td>
<td>75.1</td>
<td>30.9</td>
<td>110.8</td>
</tr>
<tr>
<td>E4</td>
<td>82.3</td>
<td>14.5</td>
<td>145.3</td>
</tr>
<tr>
<td>WLS5</td>
<td>42.4</td>
<td>10.3</td>
<td>49.3</td>
</tr>
<tr>
<td>SDP</td>
<td>39.5</td>
<td>9.7</td>
<td>46.8</td>
</tr>
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</table>
Accuracy vs. Sampling Rate

- To make the noise detectable, the number of sampling points on a waveform should be selected such that the time between two consecutive sampling points is at most as large as the crosstalk noise width

<table>
<thead>
<tr>
<th>P (# Sampling Points)</th>
<th>50</th>
<th>40</th>
<th>30</th>
<th>10</th>
<th>5</th>
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</thead>
<tbody>
<tr>
<td>Delay Error (%)</td>
<td>9.4</td>
<td>9.6</td>
<td>10.1</td>
<td>13.6</td>
<td>14.9</td>
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<tr>
<td>Run time (µs)</td>
<td>81</td>
<td>74</td>
<td>64</td>
<td>51</td>
<td>42</td>
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</tbody>
</table>
Hybrid Algorithm (SDP+E4)

- k = number of $0.5V_{dd}$ crossing points of the noisy waveform
  - if $k = 1$ Apply E4
  - else Apply SDP

<table>
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<tr>
<th>Method</th>
<th>Delay (ps)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C1</td>
</tr>
<tr>
<td></td>
<td>Max</td>
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</tr>
<tr>
<td>SDP+E4</td>
<td>39.5</td>
</tr>
</tbody>
</table>
Complexity and Runtime

• The worst-case complexity of all techniques is $O(P)$, where $P$ is the number of sampled points.

• To accomplish delay propagation through a gate on *Sun Blade 1000* machine:
  - On average P1, P2, LSF3, and E4 take about $40\mu s$ whereas WLS5 about $60\mu s$.
  - For SDP this takes around $65\mu s$ for $P = 35$. 
Application in Xtalk Target Identification

- There is an astronomical number of pairs of lines with nonzero coupling capacitance.
- Fortunately, very small number of targets can in fact produce errors.
- It is desirable to prune as many non-error producing targets as possible prior to the expensive test generation step.
Crosstalk Target Identification by SDP

- $AT_{\text{max}}(V)$: Upper bound on the arrival time increase with crosstalk
- Having the noisy input waveform, SDP is utilized to calculate accurate arrival time at the output.
- $AT_{\text{max}}(V) < R(V) \Rightarrow$ Target can be pruned.

Diagram:
- $AT_{\text{max}}(V_{\text{out}})$
- $R(V_{\text{out}})$
- $V_{\text{in}}$, $V$, $V_{\text{out}}$
- $C_m$
- $A_{\text{in}}$, $A$, $A_{\text{out}}$

AT(x): Arrival time at x
R(x): Required arrival time at x
Conclusions

• Presented an efficient method based on the sensitivity of the output to the noisy input for accurate gate delay propagation for the purpose of STA and crosstalk target identification

• Proposed a hybrid algorithm to selectively use the sensitivity-based or energy-based to further increase the accuracy

• Our techniques can be easily embedded in conventional STA tools
  - They do not need any additional cell characterizations and hence are compatible with current cell libraries
Complexity Analysis

1. All conventional gate delay propagation techniques can determine the required crossing points for the waveforms such as the $0.5V_{dd}$ crossing points in $O(P)$ time.

2. They can all apply closed form formulas to find the coefficients $a$ and $b$ for $\Gamma_{\text{eff}}^{\text{in}}$ of order $O(P)$.
   - The closed form formulas consist of several summations over $P$.

3. WLS5 has an additional step (Step 1) to calculate $\rho_{\text{noiseless}}$ which is likewise of order $O(P)$. 
Complexity Analysis of SDP

1. SDP needs to estimate $\rho^{\text{eff}}$ (in SDP-Step 2)
   
   - For each point it takes a constant time to perform part 2.a of SDP-Step 2, because P and the slew for the noiseless input is known, hence the sampling time that has a certain voltage level can be calculated without any searching.
   
   - Part 2.b of SDP-Step2 is a value assignment which can be performed in constant time.
   
   - Step 3 of SDP has complexity of $O(P)$ because Equation 4 is the summation of $P$ terms each calculated in constant time.