On the Assumption of Normality in Statistical Static Timing Analysis

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Outline

- Introduction
- PERT networks
- Normal distribution
- Max of normals
- Sum of normals
- Numerical experiments
- Conclusion

Introduction

- SSTA: the problem is difficult and approximations have to be made.

- Normality is useful in many respects.

- However, there are situations when the approximations are inaccurate.

PERT networks

- Project Evaluation and Review Techniques
  - Project with activities (e.g., tasks).
  - Tasks are random. Assumed to be independent.

- STA can be traced to PERT (and CPM).

PERT networks (cont'd)

- Analysis techniques
  - Monte Carlo
  - Propagation of cdf's and pdf's
  - Bounds
  - Markovian networks
  - Heuristics
- Complexity
  - Hagstrom (1988): independent activities, 2-value activities
    - Computation of cdf of completion time is #P-hard.
    - E[completion time] cannot be computed in polynomial time (unless P = NP).

SSTA - Challenges

- Large scale
- Complex cdf's
- Correlation
  - Reconvergent fanouts
  - Spatial: within cell, die, wafer
  - Wafer-to-wafer, lot-to-lot,…
- Must be of signoff quality

The univariate normal

Bell curve: symmetric and unimodal

If $X \sim N(\mu,\sigma)$

- $X + a \sim N(\mu+a,\sigma)$
- $bX \sim N(b\mu,|b|\sigma)$

Delays

If the channel length L is random with mean μ_L,

$$D(Co,Si,L) \approx D(Co,Si,\mu_L) + \partial D/\partial L \; (L - \mu_L)$$

If $L \sim N(\mu_L,\sigma_L)$

$$D(Co,Si,L) \sim N(D(Co,Si,\mu_L), |\partial D/\partial L| \sigma_L)$$

note: Sensitivities depend on load and slew.

Issues in delay modeling

- Delay modeled as normal: can be negative.
- Output load capacitances are r.v.'s.
- Slews are r.v.'s.

D(CO,SI,L)

- The dependency is nonlinear.

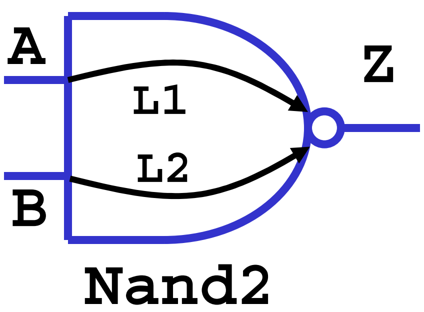
It is unlikely that gate delays are well modeled by normals.

The multivariate normal

$$X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N(\mu, \Sigma)$$

- Implies $X_1 + X_2 \sim N(\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2 + 2\sigma_{12}})$
- The covariance matrix is useful when incorporating spatial correlations.

$$\begin{pmatrix} L_1 \\ L_2 \end{pmatrix} \sim N(\mu_L, \Sigma_L)$$



The basic STA operations

- add, sub, max, and min

- We will focus on max and add.

Max of normals

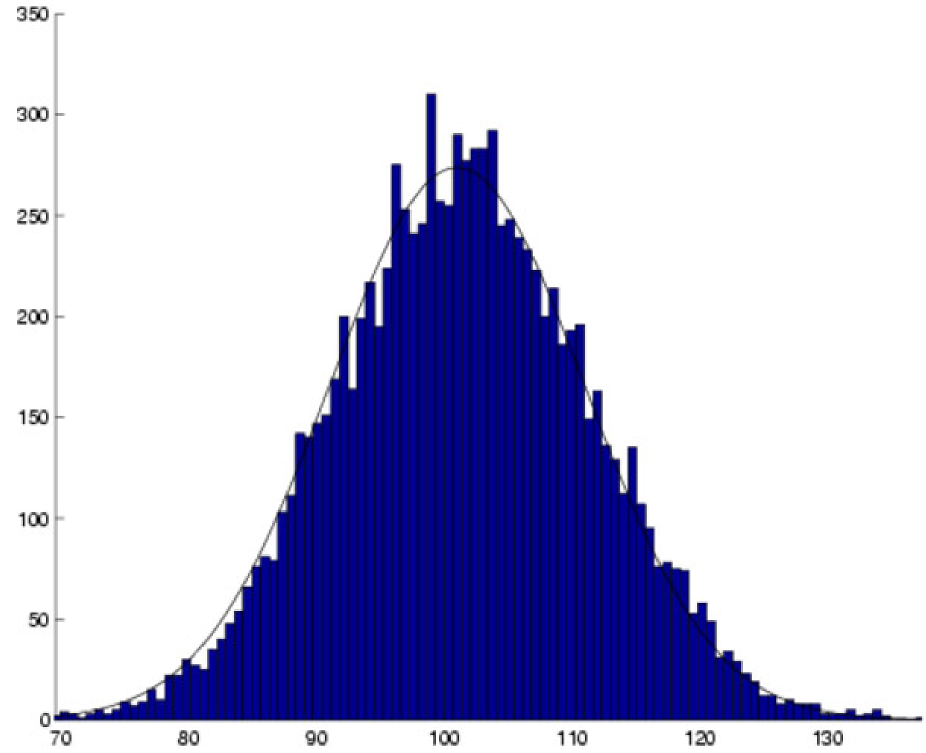
THE MAX OF TWO NORMALS IS NOT NORMAL.

- **Clark (1961):**
  - **Provides the exact first four moments of the max of two normals.**
  - **Approximations: assumes the max of two normals is normal, and iterates to find the max of three or more normals (jointly normal).**
  - **Commonly used in SSTA.**

Max: symmetric and unimodal

- dim = 10
- μ_i = 100, σ_i = 10
- Cov_{ij} = 9.95 $(i \neq j)$
- Histograms with 10,000 observations

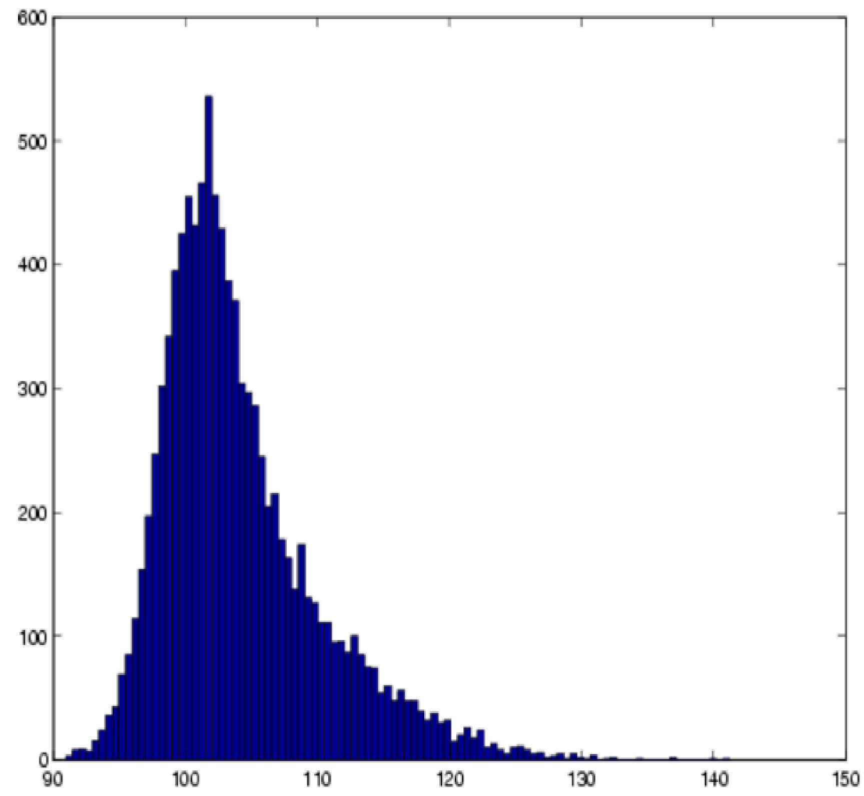
- Good fit



Max: skewed

- $\mu_1 = 100, \ \sigma_1 = 3$
- $\mu_2 = 100, \ \sigma_2 = 10$

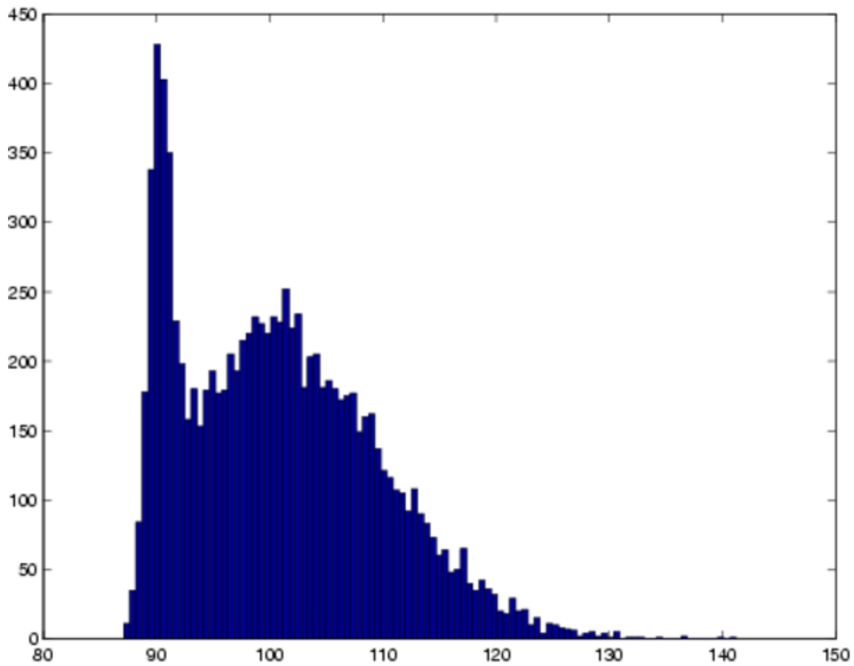
- Bad fit: skewed to the left



Max: bimodal

- $\mu_1 = 90, \sigma_1 = 1$
- $\mu_2 = 100, \sigma_2 = 10$

- Bad fit: bimodal



Sum of normals

THE SUM OF TWO NORMALS IS NOT ALWAYS NORMAL.

If A = arrival (normal) and D = delay (normal)

$\not\Rightarrow$ A + D normal

When is the sum of normals guaranteed to be normal?

- If A and D are jointly normal.
- If A and D are independent.

Normality of arrivals (sums of delays)

IT IS NOT EASY FOR ARRIVAL TIMES TO BE NORMAL.

- Theorem: **if the sum of 2 independent variables is normal, then, they both *must* be normal.**
- **Let $A_n \equiv D_1+D_2+\ldots+D_n$, D_i's independent.**
- **Hence, if a single D_i is not normal, then, A_n *cannot* be normal.**

Normality of arrivals per the Central Limit Theorem (CLT)

THE SAMPLE SIZES SEEN IN SSTA MAY NOT BE LARGE ENOUGH FOR THE CLT TO BE APPLICABLE.

- Assume the D_i's are i.i.d. From the CLT

$$A_n \approx N(n\mu, \sqrt{n}\sigma)$$

  - How fast is the convergence? Use the Berry-Esseen bounds
    - If D_i~Unif(95,105), n=100, the error in the CLT is at most 22%.
- In SSTA:
  - The D_i's are not i.i.d.
- There are CLTs for non-i.i.d r.v.'s
  - Sample sizes must be even larger.
- Logic depth is typically small (8-30).

Normal sums

In SSTA, "$A_n = D_1+D_2+\ldots+D_n$ is normally distributed" is probably the exception!

Experimental setup

- L ≡ L drawn ~ N(100,10)
  - All other parameters are at nominal
- Uncorrelated channel lengths
- Ignored interconnects
- Considered delays and slews
- Looked at arrival times (max and add)
- For each circuit, ran a Monte Carlo experiment with 1000 replicates
- Validated Monte Carlo with SPICE (small circuits)

Experimental results: Fanout tree of inv.'s, 9 levels of logic

Rise (ps)	Mean	Std Dev	95%-q	99%-q
Propagat.	284.3	3.2	288.5	291.9
Monte Carlo	289.6	8.1	300.6	309.8
Rel.Err(%)	-1.81	-60.04	-4.01	-5.78

- The means were estimated fairly accurately.
- The standard deviations were consistently underestimated.
- We can expect this to worsen with more variations and with future technologies.

Conclusion

- The assumption of normality is useful in many respects.
- However, there are situations when it is inaccurate.
- There are recent results to deal with non-normality.